Research on dynamic measurement system of sensor's transmission error based on time series prediction method

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The dynamic measurement of mechanical transmission error (shorter form "TE") has important denotations in respect to improve accuracy of transmission chain and fault diagnosis. However, high precision grating sensor is expensive and demands high levels of working environment. The time-grating displacement sensor has far more superiorities in structure, techniques and anti-interference and cost etc. Therefore, in this paper, the time-grating is taken as the angle sensor. Using the time series prediction method, the time-grating absolute angle signal is transformed into incremental signal by AR model. The calibration test suggested that the prediction error was less than ± 2 ". The whole TE system design adopted chiefly pulse subdivision counting measuring method, which has the following advantages: ①decreasing the test error effectively; ② the intelligent measurement is accomplished by combining measurement sampling, data processing and result analysis together. On the hardware design of the system, the FPGA is chosen as core, the USB module realize the fast transmission of data. Then calibrates the TE curve by mechanic phase-shifting and obtains the measuring accuracy 0.317%.

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1. Introduction

It is significant of dynamic measurement for the TE of mechanical chain, particularly where transmission of high quality is required, such as weapon system, precision-machined tool and printing machinery. The exact transmission relations can be obtained by precision measuring TE of whole mechanical chain, rather than researching process and error of single parts. The high precision part does not mean high precision transmission, which also has a close relation with the assembly relation between all parts. It has the good practical significance to give guidance for processing, matching and assemble of part by dynamic measuring the TE, providing an objective assessment and analyzing the sources of these errors. The dynamic measurement of TE usually uses grating as an angle sensor, so the grating precision will directly affect the result of TE testing. Now the domestic grating has poor precision, the common brands are HEIDENHAIN, RENISHAW et al.. The general accuracy of grating is $\pm 1''$, which can buy from market, and the purchase of higher accuracy grating is severely restricted. The angle sensor is the "heart" of numerical control equipment; if it is long-term limited by people, it will restrict the development of Machinery Industry. The time-grating displacement sensor is an original invention ^[1-5], compared with the traditional grid type of sensor, it has obvious advantage in respect of

structure, manufacturing technology, electromagnetic countercountermeasures and cost.

2. Dynamic testing principle of time-grating and TE

To establish a relative motion double coordinates with time examining point, we can convert measurement of absolute space displacement in one coordinate to measurement of relative time-lag in another coordinate ^[1], thereby realizing the time-space coordinate transformation of measuring basis in displacement measurement, and make the clock pulse with a space significance. So theoretically speaking, a new type of displacement sensor can be obtained; it can convert the demand for spatial indexing in traditional sensor into the demand for time dividing. The resolution and precision of this kind of sensor depend largely on the frequency and precision of clock pulses, therefore is called "Time-grating displacement sensor".

According to the definition of transmission error, the transmission error of mechanical chain can reflect the relative displacement of the ends of chain transmission. Let transmission ratio is $I(I \ge 1)$, theoretica -1 displacement of high-speed shaft is Φ_1 , and practical displacement is φ_1 , theoretical displacement of low-speed shaft is Φ_2 , and practical displacement is φ_2 , so the derivative process of TE as F ig.1 shows.



Fig.1 Graph of derivative process of TE

 φ_1 —practical displacement of high-speed shaft; φ_2 —practical displacement of low-speed shaft; *t*—Time; Φ_1 —theoretical displacement of high-speed shaft

Fig.1(a) shows the practical angle curve of high/low speed shaft sensor varying with time, the thicker one is the theoretical angle Φ_1 curve of high-speed shaft, which is calculated according to transmission ratio I and angle Φ_2 of low-speed shaft. Fig.1(b) takes the practical angle of low-speed shaft as horizontal ordinate. Take curve φ_1 (t) and curve Φ_1 (t) exchange in Fig.1(a) and turn into the angle curve with φ_2 as an independent variable. The Fig.1(c) shows the transmission error calculated based on Fig.1 (b). The Fig.1(d) is the discretization of the continuous curve in Fig.1(c), which is beneficial to discrete sampling.

The synchronous displacement comparison method being used in transmission error measuring [6] takes one end location as basis to compare with another end location. This is a kind of fixed space interval sampling. The time-grating is a kind of absolute position sensor with timing scanning, so its sampling value is error curve in time domain according to time divided equally. X-axis is the time t, sampling at regular intervals. In the face of the problem, the research firstly considers the thought on the basis of "sampling in time domain and analyzing in spatial domain", the detailed scheme as follows: Step1: Sample data simultaneously from high-speed shaft and lowspeed shaft respectively, obtain sampling signal based on the time divisions, as Fig.2(a) shown. Because the measured movement is a continuous process, there is a continuous curve where it is vary with time. Step 2: curve fitting in accordance with some function law, as Fig.2(b) shown. Step 3: divide continuous curve $\phi_{0}(t)$ in accordance with space equal division and obtain new discrete data ϕ_2 (curve), as Fig.2(c) shown.



Fig.2 Derivative process of TE measured by time-grating

According to the curve φ_2 , we can get new discrete data φ_1 (curve) corresponding one to one, therefore the TE data and curve in Fig.2(d) can be obtained and have the same significance as Fig.1(d).

The merit of this method is that the principle is clear and simple, namely the TE curve can be measured by two channels time-grating with synchrony trigger circuit. The weaker point is: ① Because of fixed time interval sampling but not uniform movement, so the starting point of each measure is different. Thus, the indicating value's stability of testing system can not be judged by repeatability of multiple measured curves. ② The TE can not real-time display but display after finishing measure and fitting operation. ③ The traditional practical experience^[7-10] for incremental optical grating and magnetic grating can not to be used for reference.

In order to solve the problem above, the research starts from the s ensor. Firstly absolute angle signal of time-grating can be turn - ed into incremental pulse, which can be used for realizing TE measur ement. In this way, the existing FMT (Full Micro-computeriz - ed Testing and Analysis System) can be used for measuring.

3. Time-grating signal research converting from absolute to incremental

The time-grating samples at *T* intervals to obtain absolute angle value, previous *n* measured angle values of time (from $T_{j-(n-1)}$ to T_j) are $\theta_{j-(n-1)}, \dots, \theta_{j-1}, \theta_j$, which can be considered as a time series. The time series model can be created to predict the future value; consequently the pulse is generated ^[11-12].

The prediction measuring method as shown in Fig.3, firstly using the previous *n* measured angle values $\theta_{j_{-(n-1)}}, \dots, \theta_{j_{-1}}, \theta_j$, to predict angular displacement $\Delta \theta_j$ in the next measuring period (from T_j to T_{j+1}), and generate the incremental pulse signal to indicate $\Delta \theta_j$ i n next measuring period. The prediction method can convert the absol ute discrete angle measured value into incremental continuous pulse s ignal. The time-grating angular displacement of the *j*th measuring period (from T_{j-1} to T_j) is given as follows:

$$\Delta \theta_j = \theta_j - \theta_{j-1} \tag{1}$$





Fig.3 Principle of prediction measure

Based on modeling the present and past angle displacement value $\theta_{j-(n-1)}, \dots, \theta_{j-1}, \theta_j$ of *n* sampling periods, using the time series theory, we can obtain the angle displacement predicted value of next timegrating measuring period (from T_j to T_{j+1}):

$$\Delta \hat{\theta}_{j+1} = L(\Delta \theta_{j+1} | \Delta \theta_j, \Delta \theta_{j-1}, \cdots, \Delta \theta_{j-(n-1)})$$
⁽²⁾

In that way, the number of output pulse by using Pulse-Width Modulation (PWM) to output in next measuring period (from T_j to T_{j+1}) is:

$$P_{i+1} = (\Delta \hat{\theta}_{i+1} - e_i) / Q \tag{3}$$

Where, e_j is the prediction error of the last period (from T_{j-1} to T_j), Q is pulse equivalent.

The function contains the revision of prediction error in last period, the prediction error is the difference value between previous predict ion value at T_{j-1} and this measuring value of angle increment at T_j . Through such a process, the predicting error will be compensated in the next period, so the high-precision measurement is given. Modeling the time series $\{X_i\}$ by using *p* step auto regression models AR(*p*), the function as follows:

$$X_{t} = \sum_{i=1}^{p} \phi_{i} X_{t-i} + \varepsilon_{t}, t \in \Box$$

$$\tag{4}$$

Where, $\{\varepsilon_i\}$ is white noise WN($(0, \sigma^2)$, $\phi = (\phi_1, \phi_2, \dots, \phi_p)^T$ is the auto regression coefficient of AR(p) model with $\phi_p \neq 0$. In the AR(p) model, according to $X_n, X_{n-1}, \dots, X_{n-p+1}$, using recursive prediction to predict X_{n+1} , so the optimum linear prediction expression is:

$$\hat{X}_{n+1} = L(X_{n+1} | X_n, X_{n-1}, \cdots, X_{n-p+1}) = \sum_{i=1}^{p} \phi_j X_{n-(i-1)}$$
 (5)

Make preprocessing of zero-mean-normalization for observed data $X_1, X_2, ..., X_N$:

$$Y_{t} = X_{t} - \overline{X}_{N}, t = 1, 2, \cdots, N$$

$$\overline{X}_{N} = \frac{1}{N} \sum_{j=1}^{N} X_{j}$$
(6)

Establishing a AR(p) model for data { Y_t }, then structuring the estimation of self covariance function from sample $X_1, X_2, ..., X_N$:

$$\hat{\gamma}_{k} = \frac{1}{N} \sum_{i=1}^{N-k} y_{i} y_{i+k}, k = 0, 1, \cdots, p$$
(7)

In AR(*p*) model, the moment estimation and autoregressive coefficient of WN variance: $\hat{\sigma}^2, (\hat{\phi}_1, \hat{\phi}_2, \dots, \hat{\phi}_p)^{\mathsf{T}}$ is determined by sample Yule-Walker equation:

$$\begin{bmatrix} \hat{\gamma}_{1} \\ \hat{\gamma}_{2} \\ \vdots \\ \hat{\gamma}_{p} \end{bmatrix} = \begin{bmatrix} \hat{\gamma}_{0} & \hat{\gamma}_{1} & \cdots & \hat{\gamma}_{p-1} \\ \hat{\gamma}_{1} & \hat{\gamma}_{0} & \cdots & \hat{\gamma}_{p-2} \\ \vdots & \vdots & & \vdots \\ \hat{\gamma}_{p-1} & \hat{\gamma}_{p-2} & \cdots & \hat{\gamma}_{0} \end{bmatrix} \begin{bmatrix} \hat{\phi}_{1} \\ \hat{\phi}_{2} \\ \vdots \\ \hat{\phi}_{p} \end{bmatrix}$$
(8)
$$\hat{\sigma}^{2} = \hat{\gamma}_{0} - (\hat{\phi}_{1}\hat{\gamma}_{1} + \hat{\phi}_{1}\hat{\gamma}_{1} + \cdots + \hat{\phi}_{p}\hat{\gamma}_{p})$$
(9)

After processing, the time-grating signal realizes the conversion from absolute to incremental, and the incremental time-grating pulse signal is finally received.

4. Experimental research

In order to verify the effect of incremental time-grating pulse signal, an experimental plat is especially designed. It contains numerical control rotary table, circular grating (made by HEIDENHAIN), and time-grating sensor. The numerical control rotary table is mainly supporting a revolving axle with uniform rotating and force the circular grating rotating in step with timegrating. In order to guarantee the stable revolving speed, the SIEMENS numerical control servo motor is used in rotary table.

The circular grating of HEIDENHAIN is used for calibrating the incremental pulse signal, using the circular grating of ROD880 with 36000 lines and precision of ± 1 ". In order to improve the resolution of grating signal, the output sinusoidal signal is subdivided into 100 times by HEIDENHAIN IBV660B, then each of the subdivided signal can be subdivided into 4 times again by digit circuit, the final subdivided signal has the resolution of 0.09". It can be input into counting circuit with incremental pulse signal. The signals of two channels can be counted by two counters respectively, and the two channel signals can be locked in synchronism to realize the synchronous displacement comparison. The experimental plat is shown in Fig.4.



Fig.4 Diagram of experimental set-up

The precision of absolute time-grating sensor is $\pm 1.2''$. The whole experiment processed under the condition of relative uniform rotatin g. The prediction error is less than $\pm 2''$.

5. TE test system design



Fig.5 Decimals subdivision method

Measuring TE by using the counter-type method, the measurement result contains integer part (can be counted by counter) and decimal part (can not be counted by counter directly), as Fig.5 shown. This sampling error is:

$$\Delta \phi_i = (\sum p_1 + \sigma - p_0) N_1 \tag{10}$$

Where, N_1 — p_1 pulse equivalent; σ —decimal part; p_0 —the number of each p_2 sampling signal corresponding to p_1 pulse when no TE; \sum_{p_1} —number of p_2 corresponding to p_1 integer.

In order to obtain decimal part σ , the pulse signal *P* (one or two channels) from displacement sensor is considered space scale, clock signal *P_t* is seen as auxiliary time scale. The *P_t* has no space significance before sampling. Establish the movement correspondences between *P* and *P_t* around every sampling point (the time *t_s* when *P_{2i}* coming), as shown in Fig.6.



Fig.6 Displacement time-space drawing expressed by pulse signal

In Fig.6, dotted line $\phi_1(t)$ is practical motion curve of high-speed shaft, $\phi_1(t)$ is the fitting motion curve of some discrete points on curve $\phi_1(t)$, t_s is the time of p_2 coming, then the space location can be calculated by using parabolic interpolation formula:

$$\varphi_{1}^{'}(t_{s}) = \varphi_{1}(t_{0}) + \left[\frac{t_{s} - t_{0}}{t_{1} - t_{0}} + \frac{2}{\frac{t_{2} - t_{0}}{t_{2} - t_{0}}} - \frac{1}{t_{1} - t_{0}}(t_{s} - t_{0})(t_{s} - t_{1})\right]N_{1} \quad (11)$$

Here, N_1 — p_1 pulse equicalent.

The hardware system design of TE system contains data acquisition module and data transfer module. The FPGA (Field Programmable Gate Array) is selected as core unit of data acquisition module for realizing the system miniaturization and higher-reliability. In order to make the test system more portable, the data transfer module uses USB (Universal Serial Bus) to transmit information. SOPC (System On Programmable Chip) technology is the application of System on Chip (SOC) in the field of programmable devices, it can structure processor inside FPGA which can process the much of the information of data acquisition module. The data acquisition can be achieved by structuring soft core processor NIOS II, and the Fig.7 shows the block diagram of hardware system.



Fig.7 Block diagram of hardware system

6. TE measurement of machine by time-grating

Measuring the TE of machine by time-grating, the field test is shown in Fig.8.



Fig.8 Photo of field test

The measured machine is gear-hobbing machine Y3180H produced by Chongqing machine tool factory; the machine transmitted chain is shown in Fig.9.



Fig.9 Schematic diagram of machine transmitted chain

In Fig.9 : (1) is time-grating sensor of high-speed end; (2) is timegrating sensor of low-speed end. toolbar and rotary table select change gears a, b, c, d in accordance with transmission ratio 100. The practical measure error curve as shown in Fig.10(a) and Fig.10(b) shows its frequency spectrogram.



Fig.10 Test result of TE curve

Seen from Fig.10, the first three harmonic times is first, 96th and 400th according to their amplitude. In accordance to Fig.9, the first harmonic is generated by worm gear of worktable; the 96th harmonic is generated by worm of worktable; the revolving speed of bevel gear is 4 times than toolbar and the transmission ratio of machine is 100, so the 400th harmonic is generated by bevel gear. It is clear that the main three error elements are worm gear, worm, and bevel gear. Based on the Mikhail Kalashnikov^[3] error theory of machine TE, we can know that the main of each transmission driving parts' TE is the sine(or cosine) function of rotation angle, each TE amplitude of driving parts transmits according to transmission ratio, and the error of last driving parts is the vector accumulative value of previous parts. Therefore, the precision situation of machine can be obtained by TE test curve, and the original error of driving parts can be analyzed by frequency spectrogram analysis, then it is beneficial to machine maintenance and enhance precision.

The testing machine system is verified by using mechanical phase shift method. Using this method, the system can realize selfexamining by known artificial error. The testing process is shown in F ig.11:



Fig.11 Schematic diagram of mechanical phase shift method

Where, (1)Original position of sentor's shifter lever; (2) position of sentor's shifter lever after gauge block being inserted; (3) Original indication c_1 of dial indicator; (4) indication c_2 of dial indicator after gauge block being inserted; (5) Inserted gauge block (6) machine column.

The experimental process as follows: Place a dial indicator at

somewhere of senror's shifter lever of high-speed end, the plane of dial indicator is parallel with plane of sensor, and is r apart from sensor, note the reading c_1 . The experimental curve is shown in Fig.12. Firstly, let the machine worktable normally rotates one circle, then a normal test curve can be obtained. Start self-checking when it rotates second circle and insert gauge block at the same time, write down the reading c_2 . After a while, the gauge block is taken out and the reading resets to c_1 . Corresponding to the insert of gauge block, the real-time testing TE curve shifts up a distance (shift up or shift down is decided by sensor's direction of rotation, here is shift up) comparing with the first rotating TE curve shifts down a distance and coincides with the first rotation curve.

Here, the reading difference value c_2 - c_1 is the linear truth-value l of artificial error, the known distance r is the adherent point of dial indicator and lever apart from the sensor, and then we can get

$$e = \frac{206.3 \times l/r}{I} = \frac{206.3 \times (c_2 - c_1)/r}{I}$$
(12)

According to (12), *l* is converted into arc-second value *e* of lowspeed end, *e* is the angle truth-value of artificial error. Thereinto *I* is the transmission ratio between machine worktable and toolbar axis. Extract *n* points from data f_i of first rotation and second rotation, res pectively between the time of insert and take out the gauge block, th en subtract each other one to one correspondence, and we can obtain t he test value of artificial error: $e_i = f_{2i} - f_{1i}$, at last the instrument acc uracy *E* can be get by calculate the averaging value:

$$E = \frac{1}{n} \sum_{0}^{n} \frac{e_{i} - e}{e} \times 100\%$$
(13)



Fig.12 Experimental curve of mechanical phase shift method

Take Fig.11 and Fig.12 as an example, r=145mm, write down the reading of dial indicator $c_1=0$ mm, $c_2=0.650$ mm, l=650um, transmission ratio I=100, the theoretical angle truth-value of artificial error is $e=650\times206.3/(145\times100)\approx9.2"$; From working rotating angle 80° to 280°, the data difference e_i which is obtained by comparing point by point(f_{2i} and f_{1i}) is close to e. Then e is substituted into function (1 3) to get the instrument accuracy: E=0.137%.

7. Conclusion

According to the analyses and experiment above, we can come to the following conclusion:

(1)It is feasible that time-grating signal is transformed into incremental signal by time series prediction. The accuracy of transformed sensor for measuring machine can reach 0.137%.

(2)The time-grating using incremental pulse to test TE, the real machine error situation can be accurate expressed, and the frequency spectrogram analyses of test curve correspond with the practical driving chain.

(3)According to Mikhail Kalashnikov error theory, the major error caused by higher-mode vibration will be absorbed by elastic element of driving chain, but the higher-frequency error just influence the smoothness of product, which is not belong to the TE research scope.

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