# On-machine profile measurement by multiple sensors scanning method with two kinds of algorithms

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A multiple sensors scanning system which is composed of two displacement sensors and one angle sensor was studied for on-machine profile measurement. Two algorithms based on discrete Fourier transform and least squares method respectively have been developed to reconstruct the evaluated profile. Both algorithms can separate the motion errors of the scanning stage and the zero-point errors of sensors completely in the absence of noise. Also, the uncertainties caused by the random errors of sensors were deduced. It is possible for us to select optimum parameter conditions with good uncertainties. Comparisons between two algorithms, including the restriction of parameter selection, noise suppression capability and the computational complexity, have been performed. As a result, the algorithm of least squares method seems to be superior to that of discrete Fourier transform method except the calculation speed. The on-machine experiments of measuring a reflecting mirror of a linear motor have been carried out. Both algorithms are applied to reconstruct the profile of the mirror successfully. The congruence of the evaluated results between two algorithms has been confirmed by using same measurement data.

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# NOMENCLATURE

f = evaluated profile

- L = length of evaluated profile
- N = number of discrete points on L
- s = sampling interval

 $D_1, D_2$  = interval between two displacement sensors

 $d_1, d_2$  = integer coefficient where  $d_i = D_i/s, i = 1, 2$ 

 $N_1$ ,  $N_2$  = number of sampling points in the first and second scan where  $N_i = N - d_i$ , i = 1, 2

 $m_1, m_2$  = outputs of two displacement sensors

- $m_3$  = output of angle sensor
- $u_1, u_2$  = zero-point errors of two displacement sensors
- $u_3$  = zero-point error of angle sensor
- $e_y, e_p$  = translational error and pitching error of scanning stage
- $y_1$ ,  $y_2$  = combined difference from the outputs of three sensors where  $y_i = m_2 - m_1 - D_i \times m_3$ , i = 1, 2
- $c_1$ ,  $c_2$  = combined difference from the zero-point errors of three sensors where  $c_i = u_2 - u_1 - D_i \times u_3$ , i = 1, 2

# 1. Introduction

For the ultra-precise profile measurement of large specimens such as optical component and machined surface, the scanning system with multiple sensors can be employed<sup>1,2</sup>. The probes of multiple sensors are tiled horizontally or vertically with certain intervals and mounted on a scanning stage so that the whole profile of the specimen can be scanned. To obtain the real profile from the measuring data of the multiple sensors, a reconstruction algorithm should be applied to separate the motion errors of the scanning stage and the zero-point error<sup>3</sup> of each sensor. Moreover, it is better for the algorithm to reduce the influences of the random errors of sensors. In the following, several algorithms for the multiple sensors scanning method will be introduced.

The sequential method<sup>4</sup> and the integration method<sup>5</sup> are commonly applied to the two-point scanning system. Note that an additional angle sensor is necessary to detect the pitching error of the scanning stage directly. Because the sampling interval is restricted to be equal to the interval between two displacement sensors, the sequential method can realize an error-free reconstruction with the iteration equation. However, in practice, due to the limitation of the physical size of the sensors, the lateral resolution may not be enough

as desired. The integration method allows the sensor interval larger than the sampling interval. However, this algorithm can not realize an error-free reconstruction and has a high accuracy only for the profile with long spatial wavelength<sup>5</sup>.

The least squares method<sup>6,7</sup> using matrix equation can be applied to a three or more displacement sensors system with an additional angle sensor. In this case, the outputs of sensors can provide enough information to build a series of linear equations, which own a least squares solution if the jacobian matrix has a full column rank. Therefore, the least squares method can not only realize the error-free reconstruction but also reduce the influence of the random error effectively<sup>6</sup>.

Apart from the least squares method described above, there is also a kind of method using the discrete Fourier transform in frequency domain. It has been successfully applied to a three-sensor system for the roundness measurement<sup>8</sup>. Elster applied this method to a twoangle-sensor system for profile measurement firstly<sup>9,10</sup>. In order to solve the problem of the harmonics loss, Elster also proposed a twoset of two-point method which performs scanning twice with different sensor intervals. The innovative algorithm can realize an error-free reconstruction in the absence of the zero-point errors of sensors.

The zero-point error is existent objectively and is difficult to be calibrated completely by hardware. For a two-sensor system, it is well known that the zero-point error only causes a linear deviation which has almost no influence for evaluating the profile. However, for the two sets of two-point method, the calibration of the zero-point error has not been discussed adequately<sup>11,12</sup>. In this paper, two kinds of algorithms using the discrete Fourier transform and least squares method respectively will be applied to solve the profile reconstruction of the two-set of two-point system. The paper is organized as follows: In Section 2, we will build the model of the multiple sensors scanning system. An improved algorithm based on Elster's discrete Fourier transform method is proposed. Also, the least squares method will be applied to the two-set of two-point system for the first time. In Section 3, Comparisons between two algorithms, including the restriction of parameter selection, noise suppression capability and the computational complexity, will be described. In Section 4, some experiments are carried out and the results are reported.

## 2. Reconstruction Algorithms

#### 2.1 Model of Multiple Sensors Scanning System

Figure 1 shows the schematic of multiple sensors (two displacement sensors and one angle sensor) scanning system at the sampling point *n*. Two displacement sensors are installed on a scanning stage with interval  $D_1$  to scan the profile *f* along the x-axis. An angle sensor is set up on the extra line of the motion locus of the scanning stage to detect the pitching error of the stage. If we appoint the sampling interval *s* and the number of discrete points *N*, the length of evaluated profile is  $L = (N - 1) \times s$ . The start point of scanning is where the left displacement sensor is at  $x_1$  and the end point of scanning is where the right displacement sensor is at  $x_N$ . To detect the spatial wavelength  $D_1$  of the profile and its harmonic component, a second scanning with sensor interval  $D_2$  can be considered. The sampling interval at the second scan should be kept same as the first



Fig. 1 Schematic of multiple sensors (two displacement sensors and one angle sensor) scanning system at sampling point n



Fig. 2 Two scan process with sensor interval  $D_1$  and  $D_2$  respectively. The condition  $D_2 < L - D_1$  is necessary.

scan so that the same sampling points can be scanned once more. As shown in Figure 2, A valid two scan process should satisfy the necessary condition (but not the sufficient condition) that is  $D_2 < L - D_1$ .

Assume that the vertical translational error and the pitching error of the scanning stage are  $e_y$  and  $e_p$ , two displacement sensors outputs  $(m_1, m_2)$  and the angle sensor output  $(m_3)$  can be expressed as follows:

$$\begin{cases} m_1(n) = f(n) + e_y(n) + u_1 + b_0 \\ m_2(n) = f(n+d_1) + e_y(n) + D_1 \cdot e_p(n) + u_2 + b_0, & n = [1, N_1]^{(1)} \\ m_3(n) = e_p(n) + u_3 \end{cases}$$

Where  $u_1$ ,  $u_2$ ,  $u_3$  are zero-point errors of three sensors,  $b_0$  is the distance between the x-axis and the reference line of motion locus of stage,  $d_1 = D_1 / s$  is an integer coefficient,  $N_1 = N - d_1$  is the number of sampling points in the first scan. After cancelling  $e_y$  and  $e_p$ , Eq. (1) can be simplified as follows:

$$y_{1}(n) = f(n+d_{1}) - f(n) + c_{1}, \quad n = [1, N_{1}]$$
  
where  $y_{1}(n) = m_{2}(n) - m_{1}(n) - D_{1}m_{3}(n)$   
 $c_{1} = u_{2} - u_{1} - D_{1}u_{3}$  (2)

For a similar derivation, the model of the second scan can be obtained as follows:

$$y_2(n) = f(n+d_2) - f(n) + c_2, \quad n = [1, N_2].$$
 (3)

Where  $y_2$  is a combined difference from the outputs of three sensors like  $y_1$ ,  $c_2$  is also a combined difference from the zero-point errors of three sensors like  $c_1$ .  $d_2 = D_2/s$  is an integer coefficient,  $N_2 = N - d_2$  is the number of sampling points in the second scan. The aim of reconstruction algorithm is to calculate the profile *f* according to  $y_1$ and  $y_2$ .

#### 2.2 Discrete Fourier Transform Method

The basic algorithm combining two scan data  $y_1$  and  $y_2$  to reconstruct the profile exactly was presented by Elster<sup>9</sup>. In his method, the discrete Fourier transform (DFT) method is used without considering the zero-point errors of each sensor. Yin proposed a calibration method to eliminate the influence of the zero-point errors, however a mistaken hypothesis exists in the initial derivation<sup>12</sup>. Moreover, there is not a total solution in the presence of the zeropoint errors and the random errors of sensors.

To perform DFT toward the total discrete points N, profile f should be looked as a function with a period L. By use of the natural extension method<sup>9</sup>, the values of  $y_1$  and  $y_2$  when  $n = [N_1+1, N]$  and  $n = [N_2+1, N]$  can be calculated from the known values of  $y_1$  and  $y_2$  in Eq. (2-3). By performing DFT on  $y_1$  and  $y_2$ , the following expressions can be obtained:

$$Y_{i}(k) = F_{i}(k) + a_{i}Z(k), \quad \sin(\pi d_{i}k / N) \neq 0, \quad i = 1, 2$$
  
where  $a_{i} = c_{i}/(d_{i} \times s)$   
$$Y_{i}(k) = \sum_{n=0}^{N-1} y_{i}(n)W_{N}^{nk} / W_{N}^{-d_{i}k} - 1$$
  
$$F_{i}(k) = \sum_{n=0}^{N-1} [f(n+d_{i}) - f(n)]W_{N}^{nk} / W_{N}^{-d_{i}k} - 1$$
  
$$Z(k) = \sum_{n=0}^{N-1} (s \times n)W_{N}^{nk}$$
  
$$W_{N}^{nk} = \exp(-j2\pi nk / N)$$

Eq. (4), especially for function Z, is deduced exactly and the process is omitted here. Because  $F_1$  is equal to  $F_2$  in certain common range of k, the difference of  $a_1$  and  $a_2$  can be calculated as follows:

$$a_{12} = a_1 - a_2 = (Y_1(k) - Y_2(k)) / Z(k), \qquad k \in \mathbf{U}_{12}$$
  
where  $\mathbf{U}_{12} = \{ \sin(\pi d_1 k / N) \neq 0 \cup \sin(\pi d_2 k / N) \neq 0 \}$  (5)

Using the compensation factor  $a_{12}$ , a combined Fourier coefficient can be obtained as follows:

$$Y_{c}(k) = \begin{cases} Y_{2}(k) + a_{12}Z(k), & \sin(\pi d_{1}k / N) = 0\\ Y_{1}(k), & \sin(\pi d_{1}k / N) \neq 0 \end{cases}$$
(6)

By performing the operation of Inverse Discrete Fourier Transform (IDFT) on  $Y_c$ , the profile *f* can be obtained immediately. However, to reconstruct the profile in whole spatial frequency which means k = [1, N - 1] except 0, the following conditions should be satisfied:

$$\begin{cases} \text{GCD}(d_1, d_2) = 1, \quad d_1, d_2 > 1 \\ N = p \times d_1 \times d_2, \quad p \in \text{positive integer} \end{cases}$$
(7)

Where the notation GCD means the Greatest Common Divisor. Compared with the conditions deduced by Elster<sup>9</sup>, the new conditions allow us to select sensor interval more flexibly. Especially, when the number of discrete points N is very large, for example N = 10000, it is easy to find an optimum combination of  $d_1$  and  $d_2$  which owns a good measurement uncertainty.

Next, the measurement noise added in the outputs of sensors is considered. In this case, a unique  $a_{12}$  can not be obtained anymore. But, if we assume all the noise obeys a Gaussian distribution with zero mean, an unbiased estimator of  $a_{12}$  can be obtained by use of weighted mean method as follows:

$$\hat{a}_{12} = \sum_{k \in U_{12}} \left( \left( Y_1(k) - Y_2(k) \right) / Z(k) \cdot w_a(k) \right) / \sum_{k \in U_{12}} w_a(k)$$
  
where  $w_a(k) = |Z(k)|^2 / (\delta_1(k) + \delta_2(k)),$   
 $\delta_i(k) = d_i \sum_{q=1}^{N/d_i - 1} \sin^2(\pi q d_i k / N) / \sin^2(\pi d_i k / N), \ i = 1,2$  (8)

Using the new compensation factor calculated by Eq. (8), a series of unbiased estimators of  $Y_c$  can be obtained by use of weighted mean method as follows:

$$\hat{Y}_{c}(k) = \frac{Y_{1}(k) \cdot w_{1}(k) + (Y_{2}(k) + \hat{a}_{12}Z(k)) \cdot w_{2}(k)}{w_{1}(k) + w_{2}(k)} \qquad (9)$$

$$k = [1, N - 1]$$

where  $w_1$  and  $w_2$  are defined as follows:

$$w_{1}(k) = \begin{cases} 1/\delta_{1}(k), & \sin(\pi d_{1}k / N) \neq 0\\ 0, & \text{otherwise} \end{cases},$$
$$w_{2}(k) = \begin{cases} \left(\delta_{2}(k) + |Z(k)|^{2} / \sum_{p \in U_{12}} w_{a}(p)\right)^{-1}, & \sin(\pi d_{2}k / N) \neq 0\\ 0, & \text{otherwise} \end{cases}$$

By performing the operation of IDFT, the estimator of the profile f can be obtained. The measurement uncertainty caused by the random errors from the sensors is investigated by use of the Monte Carlo method<sup>11</sup>. The basic results are that the uncertainty has nothing to do with the profile itself and the zero-point errors of sensors. The value of the uncertainty is convergent and just determined by the number of discrete points *N*, sampling interval *s* and sensor intervals  $D_1, D_2$ .

Figure 3 shows an example of uncertainties of reconstructed profile by DFT method. The number of discrete points *N* is 330 and the sampling interval *s* is 1 mm. Additional random errors of displacement sensors and angular sensor obey zero mean Gaussian distribution with standard deviations of 10 nm and 1 nm/mm respectively. Solid line is the case that  $d_1 = 10$ ,  $d_2 = 11$  and dot line is the case that  $d_1 = 3$ ,  $d_2 = 11$ . As we can see, the uncertainties of each point are different. The edge parts on the two sides of the uncertainty curves suffer much more increase than the middle parts of the curves, since the edge parts of the profile can only obtain the data of one displacement sensor in a scan process. If we ignore the edge influence, most of the values of uncertainties are close to 12 nm which owns the same level as the random errors of the displacement sensor.

#### 2.3 Least Squares Method

Compared with discrete Fourier transform method, it is much simpler to solve the model of the multiple sensors scanning system by use of least squares (LS) method<sup>6</sup>. Firstly, we define the least squares



Fig. 3 Uncertainties of reconstructed profile by discrete Fourier transform method. Solid line is the case that  $d_1 = 10$ ,  $d_2 = 11$ . Dot line is the case that  $d_1 = 3$ ,  $d_2 = 11$ . The number of discrete points *N* is 330 and the sampling interval *s* is 1 mm. Additional random errors of displacement sensors and angular sensor had standard deviations of 10 nm and 1 nm/mm respectively.

reference line of the evaluated profile as the x-axis (cf. Figure 1). Therefore, two points on the profile, for example  $f_{N-1}$  and  $f_N$ , can be calculated by the other points on the profile as follows:

$$\begin{cases} f(N-1) = \sum_{i=1}^{N-2} (i-N)f(i) \\ f(N) = \sum_{i=1}^{N-2} (N-1-i)f(i) \end{cases}$$
(10)

According to Eq. (2-3, 10), the observation equation can be obtained as follows:

$$\mathbf{Y} = \mathbf{B}\mathbf{X}$$
  
where  $\mathbf{Y} = [y_1(1), y_1(2), ..., y_1(N_1), y_2(1), y_2(2), ..., y_2(N_2)]^T$ .(11)  
$$\mathbf{X} = [f(1), f(2), ..., f(N-2), c_1, c_2]^T$$

Here **Y** denotes the column vector of the measured data in two scan process and **X** denotes the column vector of unknown parameters (not including final two points of profile *f*). **B** denotes the Jacobian matrix, the elements of which are determined by *N*,  $d_1$  and  $d_2$ . A least squares solution of **X** can be obtained when the column rank of **B** is equal to the column size of **B**, also the row size of **B** is larger to the column size of **B**. The above conditions can be simplified as follows:

$$\begin{cases} GCD(d_1, d_2) = 1, & d_1, d_2 > 1 \\ N > d_1 + d_2 \end{cases}$$
(12)

In the least squares method, the measurement uncertainty caused by the random errors from the sensors can be obtained easily using the transmission matrix as follows:

$$\mathbf{S}_{\mathbf{n}} = (\mathbf{B}^{\mathrm{T}} \mathbf{S}^{-1} \mathbf{B})^{-1}.$$
(13)

Here **S** denotes the error matrix of the measured data, the elements of which can be determined by the standard deviation of the random errors of sensors.  $S_p$  denotes the error matrix of the parameters, the diagonal elements of which show the variances of the parameters (not including final two points of profile *f*). The expressions of detail elements about **S** and  $S_p$  are omitted here<sup>6,7</sup>.

Figure 4 shows an example of uncertainties of reconstructed profile by least squares method. The number of discrete points N is 330 and the sampling interval s is 1 mm. Additional random errors of



Fig. 4 Uncertainties of reconstructed profile by least square method. Solid line is the case that  $d_1 = 10$ ,  $d_2 = 11$ . Dot line is the case that  $d_1 = 3$ ,  $d_2 = 11$ . The number of discrete points *N* is 330 and the sampling interval *s* is 1 mm. Additional random errors of displacement sensors and angular sensor had standard deviations of 10 nm and 1 nm/mm respectively.

displacement sensors and angular sensor obey zero mean Gaussian distribution with standard deviations of 10 nm and 1 nm/mm respectively. Solid line is the case that  $d_1 = 10$ ,  $d_2 = 11$  and dot line is the case that  $d_1 = 3$ ,  $d_2 = 11$ . As we can see, the feature of the uncertainty curve is similar to the discrete Fourier method (cf. Figure 3). But the effect of the edge parts on the two sides seems not so abrupt. If we ignore the edge influence, most of the values of uncertainties are close to 12 nm.

#### 3. Comparison of Two Kinds of Algorithms

As described above, it is necessary to scan the profile twice with different sensor intervals so that the lost spatial wavelength in a twopoint scan can be recovered from the other two-point scan. Both discrete Fourier transform (DFT) method and least squares (LS) method have the ability to reconstruct the profile in the presence of zero-point errors and random errors of sensors. But the procedure of calculation for DFT method can get highly complex and difficult to understand. On the other hand, LS method gives a direct solution by use of matrix equations.

Table 1 shows a comparison between DFT method and LS method. As for the parameter condition, it is necessary that  $d_1$  and  $d_2$  have no common divisor in either method. Additionally,  $N = p \times d_1 \times d_2$  for DFT method and  $N > d_1 + d_2$  for LS method should be satisfied. It is obvious that the selection of intervals ( $D_1 = d_1 \times s$ ,  $D_2 = d_2 \times s$ ) between two displacement sensors is much more flexible by use of LS method.

The uncertainty is a key evaluation index for algorithms of precision measurement. Monte Carlo method and transmission matrix are applied to DFT method and LS method respectively. Figure 5 shows an example of uncertainties of reconstructed profile. Solid line is the result of DFT method and dot line is the result of LS method. The non dimensional coefficient  $d_1$ ,  $d_2$  are set as 10 and 11. The number of discrete points N is 330 and the sampling interval s is 1 mm. Additional random errors of displacement sensors and angular sensor obey zero mean Gaussian distribution with standard deviations of 10 nm and 1 nm/mm respectively. Although the middle parts of both curves are almost equal, the edge parts on the two sides of DFT method suffer much more increase than LS method. With many

Item for comparison	Discrete Fourier transform method	Least squares method
multiple sensors scanning system	two displacement sensors and one angle sensor	
measurement method	scan profile twice with same sampling interval by changing sensor interval	
difficulty of algorithm for understanding	difficult	easy
interval between two displacement sensors	$d_1$ and $d_2$ have no common divisor,	$d_1$ and $d_2$ have no common divisor,
	and $N = p \times d_1 \times d_2$	and $N > d_1 + d_2$
uncertainty caused by random errors of sensors	good, but become bad at the edge parts	good (cf. Figure 5)
computational complexity	$O(n \times \log n)$	$O(n^3)$

Table 1 Comparison between discrete Fourier transform method and least squares method

simulation results, we can say that the capability of the noise suppression of LS method is better than DFT method.

Finally, the calculation speed of reconstruction is investigated. For DFT method, the reconstruction is fast even when *N* is larger than 10000. On the other hand, the consumption of time and memory for LS method will become unacceptable along with the increase of data size. Generally, the computational complexity of DFT method using Fast Fourier Transform is  $O(n \times \log n)$ , while the computational complexity of LS method using Matrix Multiplication is  $O(n^3)$ .

# 4. Experiments

A mirror with length of 330 mm is evaluated by use of the multiple sensors scanning system (cf. Figure 1). Figure 6 shows the experimental setup (top view) for the profile measurement. Two displacement sensors (LHG-110, resolution: 10 nm, accuracy: 100 nm) are mounted on the scanning stage along the scan direction with certain intervals. The sensor interval is 30 mm in the first scan, and 33 mm in the second scan. A laser interferometer with an angle measurement kit (ML10, resolution: 0.01 arc-sec, accuracy: 0.2 arcsec) is used. The reflector mirror of the interferometer is mounted on the scanning stage so that the pitching error of the stage can be measured. The stability experiments of sensors are performed at first. The difference of the output of two displacement sensors is below 25 nm, although there is a 100 nm drift of each displacement sensor in the test duration of 10 minutes. The stability result of the interferometer output in pitching error measurement is about 0.2 arcsec.



Fig. 5 Uncertainties of reconstructed profile. Solid line is the result of Fourier transform method and dot line is the result of least square method. The number of discrete points N is 330, the sampling interval s is 1 mm, and coefficient  $d_1$ ,  $d_2$  are 10 and 11. Additional random errors of displacement sensors and angular sensor had standard deviations of 10 nm and 1 nm/mm respectively.

The scanning stage is driven by a linear motor along the direction of the evaluated mirror. The number of points, sampling interval and time of scanning are 110, 3 mm and 3 minutes respectively. It means that the sampling points are 100 in the first scan, and be 99 in the second scan. Figure 7 shows the reconstructed results of three repeated measurements of the evaluated mirror by using DFT method, and Figure 8 shows the results by using LS method. The amplitudes of measured profile are all about 3 µm. The standard deviation of three repeated measurements is 53 nm for DFT method and 41 nm for LS method. Figure 9 shows the difference of the reconstructed results between DFT method and LS method by using the same measurement data. As we can see, both two algorithms can reconstruct the profile of the mirror with high repeatability. The differences of the middle part of the profile between two algorithms are below  $\pm 10$  nm. However, the differences of the edge part are below  $\pm 70$  nm. This feature is due to the difference of the capability of suppressing the random errors of sensors between two algorithms (cf. Figure 5).

### 5. Conclusions

In this paper, a scanning system based on two displacement sensors and one angle sensor is described for on-machine profile measurement. Two algorithms, which are discrete Fourier transform method and least squares method, are applied to reconstruct the profile from the outputs of three sensors. Both algorithms could provide exact reconstructions in the presence of zero-point errors of sensors and stage errors. The uncertainties caused by the random errors of sensors are deduced by using Monte Carlo method and the transmission matrix. It is possible for us to select optimum parameter conditions with good uncertainties. We also compare two algorithms. By use of least squares method, the selection of parameter could be more flexible and the uncertainty could be less than discrete Fourier transform method. The drawback of least squares method is the calculation speed, which will be unacceptable along with the increase of data size. An experimental multiple sensors scanning system has been constructed to evaluate a mirror with length of 330 mm. Both algorithms reconstructed the profile of the mirror successfully with repeatability of about 50 nm. The congruence of the evaluated results of the profile between two algorithms is also confirmed by using the same measurement data.

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Fig. 6 Experimental setup (top view) for profile measurement of a mirror with length of 330 mm



Fig. 7 Reconstructed results of three repeated measurements of the evaluated mirror by using discrete Fourier transform method. The standard deviation is 53 nm.



Fig. 8 Reconstructed results of three repeated measurements of the evaluated mirror by using least squares method. The standard deviation is 41 nm.



Fig. 9 Difference of the reconstructed results between discrete Fourier transform method and least squares method by using the same measurement data