Ultra-precision Measurement of Deep and Small Hole Diameter by Capacitive Probing Based on Iterative Error Reduction

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In order to reduce the error introduced by hole curvature during capacitive probing in ultra-precision diameter measurement of deep and small hole, a novel iterative error reduction method is proposed. A fixed point iteration equation is formulated based on the measurement equation, which is an implicit equation of the hole diameter, then an iteration process of the hole diameter is carried out, and the influence of the hole curvature on the characteristics of the capacitive probing sensor can be eliminated. Simulative and experimental results show that the iteration equation formulated has local convergence property, high convergent speed, and immunity to measurement noise, and the measurement precision is significantly improved from micrometer to sub-micrometer level.

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1. Introduction

A capacitive probing method has been proposed during our research on ultra-precision diameter measurement of small holes with large aspect ratio. A bilateral capacitive probing sensor is developed to make micro probing and triggering at two ends of the diameter to be measured, and macro displacement measurement of the cylindrical capacitive probe between two triggering positions is carried out with a laser interferometer. Because the probing is carried out in a non-contact way without stylus bending or contact distortion that is caused by probing force of traditional contact probing, small holes with aspect ratio up to 20:1 can be measured with micrometer precision.

However, the capacitance-gap characteristic of the cylindrical capacitive probe is influenced by curvature of the hole to be measured, which can't be ignored in high precision application. Traditionally, two methods can be used to solve this problem: 1) Dividing the measurement range of each capacitive probing sensor into several sub-range, and the capacitance-gap characteristic of the sensor for each sub-range is regarded as the same; 2) Introducing nominal value of the hole diameter to be measured as a parameter into the measurement equation, which is an implicit equation of the hole diameter to be measured [1]. However, in high precision application, the error introduced with two methods above is a major contributor to the measurement uncertainty.

In recent years, iterative error reduction method based on redundant observation and optimal estimation develops rapidly, it becomes a powerful tool to increase measurement and control precision [2~5] and is widely used in fields of machine vision and image reconstruction [6], spatial field reconstruction [7], data process and data fusion [8~9], and so on.

In order to reduce the error introduced by hole curvature during capacitive probing in ultra-precision diameter measurement of small holes with large aspect ratio, a novel iterative error reduction method is proposed in this paper, in which a fixed point equation is formulated based on the implicit measurement equation, and an iteration process of the hole diameter is carried out to eliminate the influence of hole curvature on the capacitance-gap characteristic of the capacitive probing sensor.

2. Problem description

The diameter measuring machine of deep and small holes that we developed is based on the structure of a coordinate measuring machine (CMM). To start a diameter measurement process, the attitude of the workpiece to be measured is first adjusted with a two-degree auto angle adjusting worktable, which can rotate around X axis and Y axis of the measuring machine, and axis of the hole to be measured is adjusted to be parallel with Z axis.

Then the cylindrical capacitive probe is drived with Y motor, and is located on the diameter to be measured by discriminating the deflection point of the sensor output during moving in Y direction. The capacitive probing sensor carried out probing and triggering at each end of the diameter. The displacement of the probe between two triggering position is measured by a laser interferometer, and the micro-gap between the cylindrical probe and the hole sidewall when triggering is measured by the capacitive probing sensor itself. The diameter of the cylindrical capacitive probe is calibrated beforehand. Supposing the probe diameter is d, the probe displacement between two triggering positions is S, and the micro-gap between the cylindrical probe and the hole sidewall when triggering is δ, then the measurement equation of the system can be expressed as

$$D = 2\delta + d + S \tag{1}$$

The capacitive probing sensor has two rectangular window shape capacitive plates which are symmetrical on opposite sides of the cylindrical probe and serve as sensing units. Figure 1 is a section of the cylindrical probe and the hole to be measured. It can be seen that the sensor has bilateral sensing characteristics and ability.

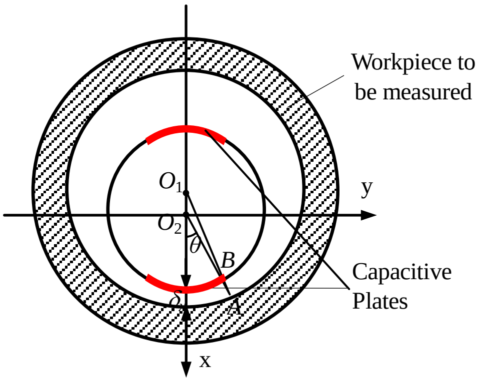

Fig. 1 Section of the cylindrical probe and workpiece

Supposing the dielectric constant of air is ε, the radius of the probe is r, the height of two rectangular capacitive plates is l, the central angle of each plate is 2ξ, then the capacitance between the probe and the workpiece during probing can be modeled based on infinitesimal method as below

$$C = \int_{-\xi}^{\xi} \frac{\varepsilon \cdot r \cdot l}{\sqrt{\cos^2\theta \cdot \{D/2-(r+\delta)\}^2 + D(r+\delta) - (r+\delta)^2} - \cos\theta \cdot \{D/2-(r+\delta)\} - r} \mathrm{d}\theta \tag{2}$$

The variety of probing capacitance C is converted to the frequency variety of the output signal of a frequency modulated (FM) circuit. The frequency of the output signal can be expressed as

$$f = \frac{1}{2\pi\sqrt{L(C_s + C)}} \tag{3}$$

Where C_s is the sum of presetted offset capacitance and stray capacitance. It's can be seen that f is a dualistic function of probing gap δ and hole diameter D, and capacitance-gap characteristics of the probing sensor is influenced by the curvature of the hole to be measured. This influence can't be ignored to guarantee model precision. The precise relation between probing gap δ, output frequency f and hole diameter D can be obtained by calibration, it can be expressed as

$$\delta = g(f, D) \tag{4}$$

According to equation (4) and equation (1), the expression below can be obtained

$$D = 2g(f, D) + d + S = \Phi(f, d, S, D) \tag{5}$$

It's clear that equation (5) is an implicit equation of D and this implicit equation can not be analytically solved. Traditionally the nominal value of diameter D is introduced into the right side of the measurement equation, and an error is introduced thereof. Supposing the nominal value of diameter D is D_E, and its true value is D_R, then

$$D_R = 2g(f, D_R) + d + S \tag{6}$$

Ignoring other error resources and noise, the error introduced by D_E can be expressed as

$$E = 2g(f, D_E) + d + S - D_R \tag{7}$$

The following expression can be obtained from Taylor expansion of equation (5) at point $D = D_R$

$$D = D_R + \frac{\partial\Phi}{\partial D_R}(D - D_R) + \frac{\partial\Phi}{2!\partial D_R}(D - D_R)^2 + \frac{\partial\Phi}{3!\partial D_R}(D - D_R)^3 + \cdots \tag{8}$$

$$E = \frac{\partial\Phi}{\partial D_R}(D_E - D_R) + \frac{\partial\Phi}{2!\partial D_R}(D_E - D_R)^2 + \frac{\partial\Phi}{3!\partial D_R}(D - D_R)^3 + \cdots \tag{9}$$

Ignoring high order terms of the Taylor expansion

$$E = \frac{\partial\Phi}{\partial D_R}(D_E - D_R) \tag{10}$$

It can be seen that the error introduced by D_E propagates in the measurement equation as a first order term; consequently a relatively large error is introduced. Supposing ε=8.854×10^{-12}, r=1.4mm, l=2.5mm, ξ=45°, L=46μH, C_s=34.7pF, the probing capacitance is 1pF, and the range of probing gap is 4μm, the simulation result when D_R=ϕ4mm, D_E-D_R=-50μm~50μm is shown in figure 2. The simulation result is shown in figure 3 when other parameters remain the same except δ=25μm, D_R =ϕ3.5mm~ϕ4.5mm. It can be seen that when D_E-D_R= -50μm~50μm, the error introduced is in micrometer level.

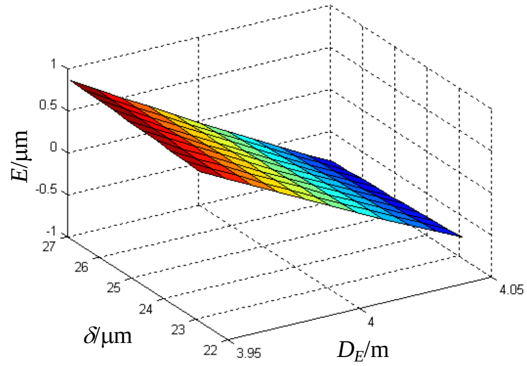

Fig. 2 Simulation result when D_R=ϕ4mm, δ=22μm~26μm

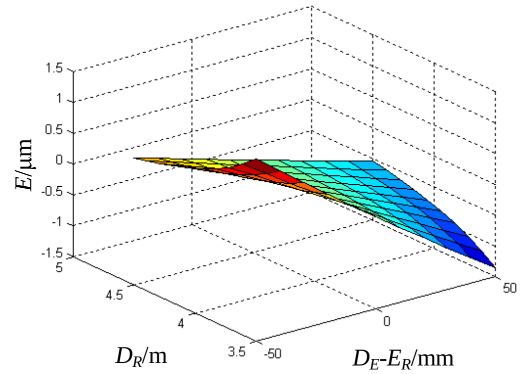

Fig. 3 Simulation result when δ=20μm, D_R = ϕ3.5mm~ϕ4.5mm

3. Iterative error reduction method

To eliminate the influence of the hole curvature to the characteristics of the probing sensor and the error introduced thereof, a iterative error reduction method is proposed. With the large volume characteristic data obtained by calibration and modeling, the error introduced can be eliminated through an iteration process. According to measurement equation (5), a single point iterative equation can be constructed and expressed as below

$$D_{i+1} = \Phi(f, d, S, D_i) \quad i = 0,\ 1,\ 2,\ \cdots \tag{11}$$

During data processing, f and S are indirect measurement parameters that have been obtained already, d has been obtained by high precision calibration, so f, S and d can be regarded as constants and equation (11) can be expressed as

$$D_{i+1} = \Phi(D_i) \quad i = 0,\ 1,\ 2,\ \cdots \tag{12}$$

If the iterative series $\{D_i\}$ converge to D_R, which is the ture value of hte hole diameter to be measured, then the problem of error reduction can be converted to a problem of root solving of a fixed point equation, and the iterative error reduction method proposed in this paper is feasible.

To prove the convergence of single point iteration equation (11), the measurement equationmight as well be regarded as the concrete hole diameter measuring process, then it's can be concluded that precision of the measurement result is higher than the nominal value of the hole diameter, which can be expressed as

$$|\Phi(f, d, S, D_E) - D_R| < |D_E - D_R| \tag{13}$$

The left and right sides of equation (13) are respectively errors of measurement result $\Phi(f, d, S, D_E)$ and nominal value of hole diameter D_E, comparing to true value of hole diameter D_R.

Parameters f, S and d can be regarded as constants. Considering the universality of D_E, equation (13) can be expressed as

$$|\Phi(D) - D_R| < |D - D_R| \tag{14}$$

$$\frac{|\Phi(D) - D_R|}{|D - D_R|} \le L < 1 \tag{15}$$

Calculate the limit of equation (15) at point $D \to D_R$, according to the isotonicity theorem around limit point

$$\lim_{D \to D_R} \frac{|\Phi(D) - D_R|}{|D - D_R|} \le L < 1 \tag{16}$$

$$|\Phi'(D_R)| \le L < 1 \tag{17}$$

According to the convergence theorem of sigle point iteration, if $\Phi(D)$ has continuous one-order derivative in the neighborhood of D_R, which is the root of equation $D = \Phi(D)$, and $|\Phi'(D_R)| < 1$, then the iteration equation $D_{i+1} = \Phi(D_i) \quad i = 0,\ 1,\ 2,\ \cdots$ has local convergence property. According to equation (17), $|\Phi'(D_R)| < 1$ has been satisfied. For the real hole diameter measuring machine, because of the characteristic continuity of real devices and parts, $\Phi(D)$ usually has continuous one-order derivative in the neighborhood of D_R, and so the local convergence of single point iteratative equation (11) is proved.

By the way, according to theory of single point iteration, the speed of convergence is decided by L. When L is more less than 1, the higher the convergent speed is. For the real hole diameter measuring machine, the precision of the measurement result is usually much higher than the nominal value of hole diameter, which means $|\Phi(D) - D_R| << |D - D_R|$. According to equations (14) and (15), L<<1, and so single point iterative equation (11) has high convergent speed.

In real application, the nominal value of the hole diameter to be measured can be used as D_0, or a coarse measurement result of a low precision measurement method can be used. A precision control parameter ξ can be used. when $|D_{i+1} - D_i| < \xi$, the iteration can be finished and D_{i+1} is chosen as the final measurement result of the hole diameter.

4. Simulation and experiment

In order to verify the iterative error reduction method proposed in this paper, supposing ε=8.854×10^{-12}, r=1.4mm, l=2.5mm, ξ=45°, δ =25μm, and D_R = ϕ4mm, choose D_R-50μm and D_R+50μm as D_0 respectively, the simulative results are shown in figure 4 and figure 5. It can be seen that the error of D_i relative to the true value of hole diameter D_R, become smaller quickly with the progress of the iteration process,, and D_i finally converges to D_R. The simulation results show that the influence of hole curvature to characteristics of the capacitive probing sensor can be eliminated, and the measurement precision can be significantly improved. The method has high convergent speed, and the precision of the iteration result become satisfactory with 4~5 times iteration.

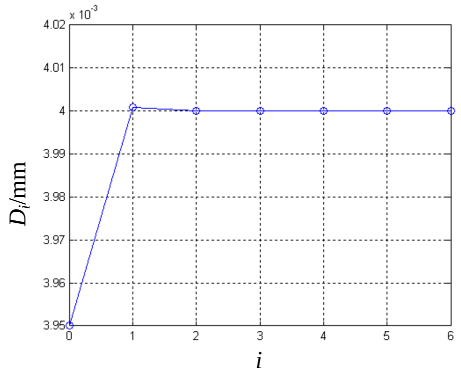

Fig. 4 Simulation result of iteration method when D_0=D_R-50μm

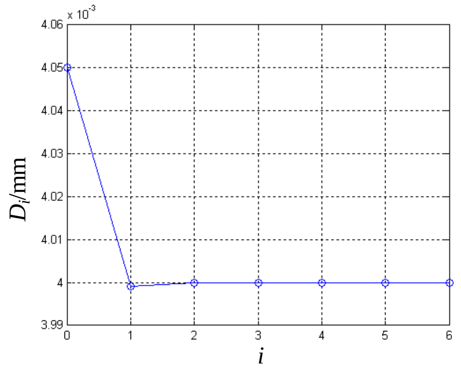

Fig. 5 Simulation result of iteration method when D_0=D_R+50μm

In order to further verify the iterative error reduction method proposed, experiments are carried out on the hole diameter measuring machine we developed. The design parameters of the capacitive probing sensor are as same as parameters used in simulation above, the probing gap is 24μm~30μm, and the probing capacitance is about 0.8pF。The workpiece to be measured is a ring gauge that has been calibrated by National Institute of Metrology P.R.China, the nominal value of its diameter is ϕ4.0mm, and the calibration result is ϕ4.2052mm. If the nominal value of the ring gauge diamter is introduced into the measurement equation,the measurement result is ϕ4.20921mm , whose precision is far from satisfactory. The experimental result with the iterative error reduction method proposed in this paper is show in table 1. The initial values of D_0 are chose as D_R+5μm, D_R+50μm and D_R+500μm respectively. Experimental results show that the measurement precision is significantly improved, and the feasibility and validity of the iterative error reduction method is proved.

5. Conclusion

During high precision diameter measurement of small holes with large aspect ratio by capacitive probing, considerable error is introduced because the hole diameter itself is a parameter in the implicit measurement function, and nominal value of it has to be used. In order to solve this problem, a novel iterative error reduction method is proposed in this paper. A fixed point equation is formulated based on the measurement equation, and the influence of the hole curvature to the characteristics of the capacitive probing sensor can be eliminate through iteration process.

Simulative and experimental results show that the iterative error reduction method proposed is simple and robust to measurement noise, and the local convergence property and high convergent speed of the method is proved. The error introduced by the nominal value of the hole diameter is reduced to a satiafactory degree with 4~5 times iterations, and the measurement precision is significantly improved from micrometer to sub-micrometer level, without additional hardware cost. The method can also be applied in other indirect measurement occasitions that demand high measurement precision.

Calibration value D_R	Initial value D_0	1st iteration D_1	2nd iteration D_2	3rd iteration D_3	4th iteration D_4
4.2052	4.21020	4.20526	4.20535	4.20535	4.20535
	4.25520	4.20423	4.20511	4.20510	4.20510
	4.70520	4.19665	4.20540	4.20525	4.20526

Table. 1 Experimental result of iterative error reduction, the unit is mm for all value

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