A study of task specific uncertainty for least square based form characterization of ultra-precision freeform surfaces

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Abstract: *In the measurement of freeform surfaces, least square based form characterization methods are widely used to access the quality of machined freeform surfaces. Although many methodologies have been proposed to improve the efficiency of the characterization process in the recent years, relatively little research have been conducted in the analysis of associated uncertainty in the characterization results which may be attributed from various sources such as measurement error, sampling strategy being used, etc.. As a result, this paper presents a study of the estimation of task specific uncertainty in least square based form characterization of ultra-precision freeform surfaces. That is, the associated uncertainty in the characterization results is estimated when the measured data is extracted from a specific surface with specific sampling strategy. Instead of estimating the whole uncertainty of a measurement result, the proposed study is more focused on the evaluation of the uncertainty caused by the misalignment of coordinate systems of measured data and design surface. Three factors are considered in the present study which include measurement error, surface form error and sample size. The results provide an important means for estimating the uncertainty of the form characterization for a specific freeform surface measurement.*

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1. Introduction

Ultra-precision freeform surfaces are widely used in advanced optical systems due to their superior optical properties [1]. These surfaces can be fabricated by ultra-precision freeform machining technologies with form accuracy in micrometer to sub-micrometer range and surface finish in nanometer level. However, form characterization of the machined ultra-precision freeform surfaces is still a challenging task due to the geometric complexity and high precision requirement.

In general, the process of the form characterization of ultra-precision freeform surfaces can be divided into two parts, i.e. data acquisition and data processing [2]. In the first part, precision 3D measuring instruments are used to extract data points from machined freeform surfaces with the guidance of appropriate sampling strategy. In the second part, the measured data are used to compare with design surface so as to evaluate the form error of the measured surface. Since the coordinate system of measured data is normally different from the coordinate system of the design surface, surface matching (corresponding search) method is required to precisely align the measured data with the design surface so as to eliminate the error caused by the misalignment of coordinate systems. Recently, many methodologies have been developed to improve the efficiency and accuracy of the surface matching process from rough matching to fine matching [3-12]. In an overview of the published literature, the least square based method is still the most widely used method in freeform surface matching.

Theoretically, the least square based method is able to match two freeform surfaces with any desired accuracy when there is no deviation between them. However, the measured surfaces always contain errors resulting from the error of measurement instruments and the form error of machined workpiece. This will seriously affect the reliability and accuracy of the sampling strategy and surface matching process. This led to uncertainty for the characterization results. The uncertainty due to the sampling strategy may be reduced by increasing the number of sample points. However, the inaccurate surface matching will lead to misalignment of the coordinate systems of two matching objects which will cause serious error to the results of the surface characterization. This is fatal for the measurement of ultra-precision freeform surfaces which require form accuracy down to sub-micrometer level. Hence, it is obligatory to analyze the uncertainty of the form characterization method so as to access the reliability of the characterization results.

Current research on the uncertainty analysis in geometric measurement is still focused on simple geometries such as circle, sphere and cylinder [13-15], and relatively little research has been conducted on the measurement of freeform surfaces. Some researchers attempt to represent the freeform surfaces by assembling simple geometries such as sphere and cylinder so that the problem is converted to analyze each assembled simple geometry [16]. Unfortunately, the method is not suitable for all types of freeform surfaces. The general model described by the Guide to the expression of uncertainty in measurement [17] is also difficult to be applied to freeform surface measurement process since the uncertainty varies with the surfaces being measured and the form characterization method being used.

As a result, this paper presents a study of the estimation of task specific uncertainty for the least square based form characterization of ultra-precision freeform surfaces. In other words, the associated uncertainty in characterization results is estimated when the measured data is extracted from a specific surface with specific sampling strategy [14]. Instead of estimating the whole uncertainty of the measurement result, the proposed study is more focused on the evaluation of the uncertainty caused by surface matching method. Three factors are considered in the study including measurement error, surface form error and sample size. The dependence of the associated uncertainty in the characterization results on the three factors is demonstrated by Monte Carlo method [17] for a designed sinusoidal surface.

2. Uncertainty of least square based form characterization

2.1 Least square based form characterization

Least square based form characterization is an optimization process to search for an optimal Euclidean motion for the measured data so that it is aligned with a template surface as close as possible and the deviation of the measured data from the design surface is considered as the form error of the measured surface. The objective function of the least square based method is given by Eq. (1) [11]:

$$F = \sum_{i=1}^{N} \left| \boldsymbol{P}_i - \boldsymbol{T}\boldsymbol{Q}_i \right|^2 \quad (1)$$

Where $\boldsymbol{Q}_i$ is measured point and $\boldsymbol{P}_i$ is the corresponding point on the design surface; coordinate transformation matrix $\boldsymbol{T}$ is a function of spatial rotation and translation; $|\cdot|$ is the distance between ($\boldsymbol{P}_i$, $\boldsymbol{Q}_i$) and is positive if $\boldsymbol{Q}_i$ is above the design surface, otherwise negative. Hence $|\boldsymbol{P}_i - \boldsymbol{T}\boldsymbol{Q}_i|$ is the form error of the measured surface at $\boldsymbol{Q}_i$. However, the convergence domain of this method is very small and the optimization may be trapped at a local minimum or even become divergent if the relative position of the measured data and the design surface is not properly supplied. Since there are many rough matching methods presented in the archived literature which can perform surface rough matching between the measured data and the design surface [5-10], it is assumed in this study that the initial position between the measured data and the design surface is already relatively close to each other so as to ensure the convergence in surface matching.

2.2 Uncertainty evaluation model

From Eq. (1), it is inferred that the uncertainty associated in the form characterization results may be resulting from many factors including the form error of the workpiece, the measurement error and the number and distribution of the measured points, and the propagation of these errors may also be different from the surface being characterized. Hence, it is very difficult to establish a universal model to analyze the uncertainty for all types of freeform surfaces. As a result, a task specific uncertainty estimation method is proposed in this study. Fig. 2 shows a framework of the task specific uncertainty estimation model. In the model, the uncertainty associated in the characterization results is estimated when the measured data are extracted from a specific surface with specific sampling strategy.

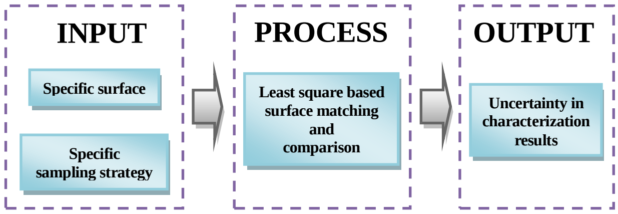
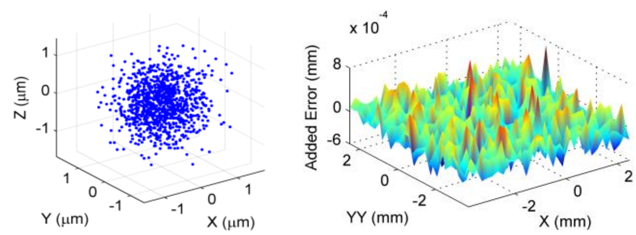


Fig. 1 Framework of uncertainty evaluation model

The model is composed of three parts, i.e. input, process and output. In the first part, the design surface of the workpiece is input into the model by adding a random form error. Then a certain number of points are sampled from the surface based on a given sampling strategy and the error of the measurement instrument is also added on each sampled data. In the second part, the least square based method is used to perform surface matching and comparison so as to evaluate the deviation of the sampled points from design surface. In the third part, the uncertainty associated in the form characterization results are estimated by comparing the characterization results with the added random form error which input in the first part.

In the model, the error of the measuring instrument and the form error of the workpiece are still unknown knowledge which must be defined and quantified. Since much effort has been applied to calibrate and compensate the systematic errors of the coordinate measuring instruments in recent years [14], the error of a well calibrated measuring instrument can be modeled as multivariable random noise so that the measurement error in an arbitrary sample point can be regarded as a sample from the distribution. Fig. 2a shows the points drawn from multivariable Gaussian distribution (MGD) and Fig. 2b shows the 3D error of measured data caused by the measurement error.



(a) points drawn from MGD (b) 3D noise error map

Fig. 2 Measurement Error generated by Multivariable Gaussian noise

The form error of the workpiece is surface variation caused by imperfect manufacturing which could be any scales with any topologies. In frequency domain, form error can be considered as random signal with low frequency. Hence, Fractional Brownian Motion (FBM) is used to model the form error of measured workpiece in this study. FBM is a continuous-time Gaussian process $B^H(t)$ on $[0\ \ T]$, which starts as zero, has expectation of zero for all t in $[0\ \ T]$, and is given the covariance function [18] as follows:

$$E\left[B^H(x)B^H(y)\right] = \frac{1}{2}\left(|x|^{2H} + |y|^{2H} - |x-y|^{2H}\right) \quad (2)$$

Where $x, y \in [0\ T]$; H is a real number in $[0\ 1]$, called the Hurst index which can be used to control the 'roughness' of the generated signal. Fig. 3 shows the surface generated with Hurst index 0.2, 0.5 and 0.8. It is interesting to note from the figures that the generated surface exhibits strong low-frequency component and has irregular behavior when the H is 0.8. This is well match with the properties of the form error of the workpiece. Hence, the Fractional Brownian motion with Hurst index 0.8 is suitable to model the form error of the machined workpiece.

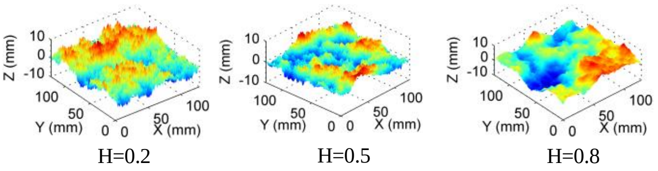


Fig. 3 Surface generated by FBM with different Hurst index

3. Experiment

The established uncertainty analysis model is validated on a designed freeform surface given by

$$z = \sin(0.5x) + \cos(0.5y) \quad -2\pi \le x \le 2\pi,\ -2\pi \le y \le 2\pi \quad (3)$$

with uniform sampling strategy. Uniform sapling strategy may not be the best sampling method. However, it is widely used in practice. In the following study, a certain number of points are sampled on the designed surface with uniform sampling strategy as the measurement data and they are transformed with rotation [1 1 1] (degree) and translation [0.5 0.5 0.5] (mm) as the initial position, i.e. indicating the misalignment of the measured data and design surface. Three factors are considered including measurement error, surface form error and sample size. Two surface parameters, i.e. pick-to-valley height S_t and root mean square error S_q are introduced and determined as follows:

$$S_t = \left|\max(|\boldsymbol{P}_i - \boldsymbol{TQ}_i|) - \min(|\boldsymbol{P}_i - \boldsymbol{TQ}_i|)\right| \quad (4)$$

$$S_q = \sqrt{\frac{1}{N}\sum_{i=1}^{N}|\boldsymbol{P}_i - \boldsymbol{TQ}|_i^2} \quad (5)$$

where N is total number of sampled points.

3.1 Uncertainty in ideal case

Fig. 4 shows the evaluated form error of the measured data in ideal noise free case, i.e. without adding measurement noise and form error. Since the measured data is directly sampled from design surface, the form error should be zero. Hence, the evaluated form error is considered as systematic error of the method used in form characterization process. It is interesting to note that the systematic error of the least square based form characterization method is extremely small down to 10^{-14} mm level when there is no measurement noise and form error contained in the measured data.

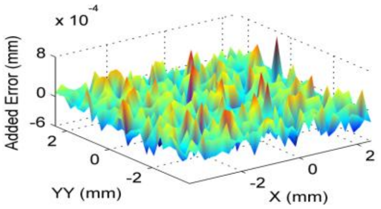


Fig. 4 Uncertainty of form characterization result in ideal case

3.2 Uncertainty caused by measurement error

The uncertainty analysis is conducted with different scales of measurement noise and different number of sampled points. A total 9 cases are studied as shown in Table 1. In each case study, 1000 Monte Carlo trials are made based on multivariable Gaussian noise with given standard deviation. Although there is no requirement by the model, the associated uncertainties in all three axes are assumed to be uncorrelated and axis-isotropic for simplicity.

Table 1 Added measurement error and number of sampled points

Measurement error	*Number of sample points*		
Mean 0 Std: 0.2 µm	30×30	60×60	90×90
Mean 0 Std: 0.5 µm	30×30	60×60	90×90
Mean 0 Std: 0.8 µm	30×30	60×60	90×90

Fig. 5 and Fig. 6 show the mean and standard deviation (Std) of the error of the estimated surface parameters due to inaccurate surface matching in surface comparison. It is found from the results that the error of the evaluated surface parameters (both St and Sq) decreases with increasing number of sampled points. Although a large scale of noise causes large error to the surface parameters, the difference also decreases with increasing number of sampled points. This may due to the fact that the noise contained in the sampled data has zero mean and this is well match with the advantage of the least square based method so that the measured data above the design surface will counteract with those under design surface in surface matching process. As a result, the effect of the measurement noise to the surface matching is reduced with the increasing number of measured data. It is interesting to note that the uncertainty associated in the least square based method is in nanometer level when the measurement noise is in sub-micrometer level. Least square based method shows its high accuracy and low sensitivity to the random measurement noise.

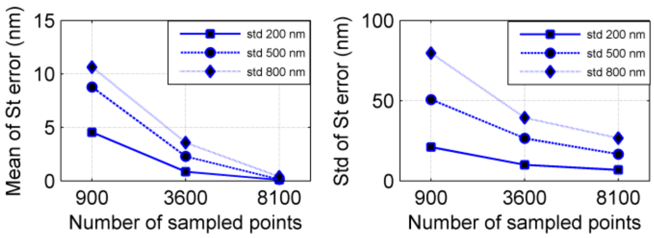


Fig. 5 Error of St of evaluated form error

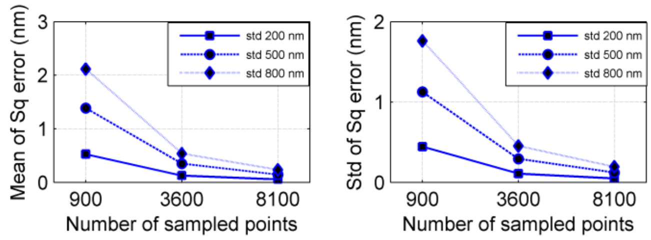


Fig. 6 Error of Sq of evaluated form error

3.3 Uncertainty caused by form error of workpiece

The uncertainty analysis is conducted with different scales of form error and different number of sampled points. The scale of random form error is described by St (Pick-to-valley height). A total of 25 cases are studied as shown in Table 2. In each case study, 1000

Monte Carlo trials are made based on the random form errors generated by Fractional Brownian Motion with given scale.

Table 2 Added form error and number of sampled points

Form error	*Number of sample points*				
1.5≤St≤2.5 μm	30×30	45×45	60×60	75×75	90×90
3.5≤St≤4.5 μm	30×30	45×45	60×60	75×75	90×90
5.5≤St≤6.5 μm	30×30	45×45	60×60	75×75	90×90
7.5≤St≤8.5 μm	30×30	45×45	60×60	75×75	90×90
9.5≤St≤10.5 μm	30×30	45×45	60×60	75×75	90×90

Fig. 7 and Fig. 8 show the mean and standard deviation (Std) of the error of the estimated surface parameters due to the misalignment of coordinate systems in surface comparison. It is interesting to find from the results that the error of estimated surface parameters is insensitive with the number of sampled points while it increases with the increase of the scale of added form error. The results show that the accuracy of the least square based method is seriously affected by the form error of measured surface and the error of the estimated St is up to 10%. 95% confidence intervals of surface parameters are shown in Fig. 9. An interval with width around 25% of St of the added form error is required to cover 95% of the error of evaluated surface parameters St and Sq. The results also infer that the form characterization results obtained by least square method are very possibly smaller than the true form error of measured surface.

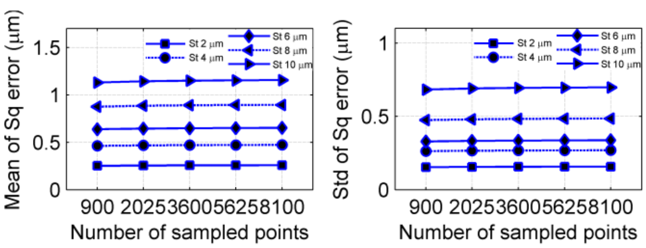


Fig. 7 Error of Sq of evaluated form error

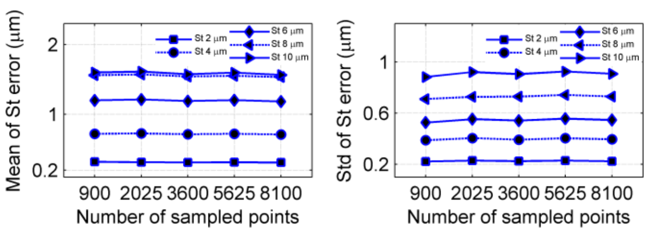


Fig. 8 Error of St of evaluated form error

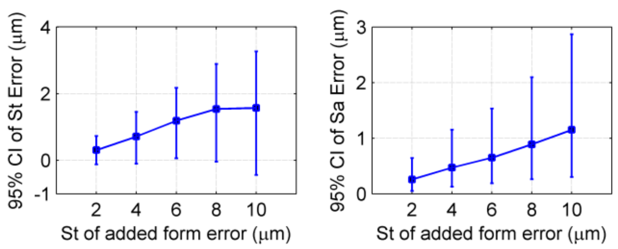


Fig. 9 95% CI of estimated parameters error

3.4 Test

Based on the previous work, cover regions for the form characterization results are estimated with certain confidence. Fig. 10 and Fig. 11 show the 95% confidence interval of the estimated Sq and St with respect to the St of the form error of the workpiece. To verify the validity of the estimated confidence interval, a total 2690 tails are generated based on the model presented in Section 2. Form error of the workpiece is randomly generated by the FBM with $Std \in [1.5\ 10.5]$ μm; Sample points are randomly generated by uniform sampling strategy with number in [900 8100]; Measurement noise is randomly generated by MGN with $Std \in [0.2\ 0.8]$ μm. For each trail, the sampled points are characterized by the least square based form characterization method. Then the error of each evaluated St and Sq in each trail are pointed on corresponding region with respect to the St of the form error generated in that trail. Fig. 10 and Fig. 11 show the test results of St and Sq respectively. For St, a total 150 trails are located outside of the cover region and the validity of the cover region is 95.9%; for Sq, total 109 trails are located outside of the cover region and the validity of the cover region is 94.4%. It is interesting note that the results well match with the given confidence level of the region.

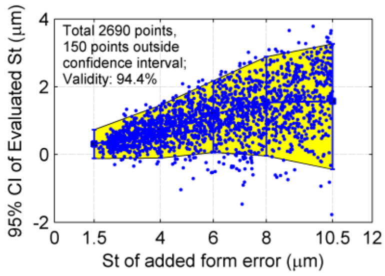


Fig. 10 95% CI of evaluated St

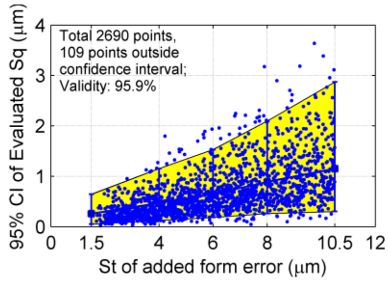


Fig. 11 95% CI of evaluated Sq

4. Conclusions (Times New Roman 10pt)

This paper presents a model to study the task specific uncertainty in the least square based form characterization of ultra-precision freeform surfaces. Three factors including measurement error, sample size and form error of workpiece are considered in the uncertainty analysis model. The measurement noise is modeled by multivariable random noise while the form error of the workpiece is generated by Fractional Brownian motion. The dependence of the associated uncertainty in the characterization results on the three factors is demonstrated by Monte Carlo method. Some results are realized as follows

(1) Least square based method is insensitive to the present of random measurement noise in the measured data. The more the sampled data, the less uncertainty contributed by the measurement noise.
(2) With uniform sampling strategy, the uncertainty contributed by form error of the measured surface is insensitive to the number of sample points. Hence, the sample size should be chosen by considering more on the reliability of the sampled points in describing the form of the measured surface.
(3) The accuracy of the least square based method is adversely affected by the form error of the measured surface. In the present

example, the error of the estimated St is up to 10%. An interval with width of around 25% of the St of the form error of measured surface is required to cover 95 % of the error of characterization results.

(4) The estimated form error of the measured surface by least square method is very possibly smaller than the true form the error of measured surface.

The present study infers that there is notable uncertainty associated in the least square based form characterization method which must be analyzed and quantified to access the reliability of the characterization results. The proposed model presents a possible way to estimate the associated uncertainty in the least square based surface matching with respect to the scale of the estimated form error. The study can be used to estimate the uncertainty of the form characterization results for a specific freeform surface measurement. For the future work, the model will be further developed to study the uncertainty in freeform surface measurement and form characterization.

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