# Optimum design of active scanning probe using parallel link mechanism 

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#### Abstract

A 3-dof active scanning probe using a translational parallel mechanism (TPM) is proposed. In this paper, the characteristics of the mechanism, optimum design, prototyping, and accuracy of the TPM are introduced. An optimum TPM design is proposed as the manipulability of the mechanism is equal in all directions. This implies that the positional sensitivities of the TPM share an isotropic relationship. The working range and the positional resolution of the mechanism are designed as 1.6 mm cubic volume and $1 \mu \mathrm{~m}$, respectively. An experimental mechanism using the proposed design is introduced. In order to evaluate the accuracy of the TPM, we have developed an experimental system for the measurement of the position of the TPM by using image processing. By applying mechanical calibration, the accuracy of the absolute position of the TPM has become $3.4 \mu m$.


## 1. Introduction

A 3-dof active scanning probe using a translational parallel mechanism (TPM) is proposed. Conventional research on TPMs for coordinate measurement has focused on fine-positioning devices working in a few dozens of centimeters in size ${ }^{1,2}$. For the actual application of coordinate measurement, it is important to study the sensing devices attached to the tips of a coordinate measuring machines (CMMs). The TPM proposed in this study is not designed for conventional positioning devices; it employs a small touching force control and can be used for a probing device working in a few millimeters in size. The TPM is attached to the tips of CMMs or numerical-control machine tools and used as a scanning probe. It has the advantages of short measurement time and accurate largecoordinate measurement. Moreover, mass of the TPM's moving part is smaller than that of conventional scanning probe with serial link mechanism. These characteristics are advantages for fast scanning of complicated shapes on wokpieces.

We have already developed a TPM with three legs $3-\underline{P} U U$ ( $\underline{P}$ : Prismatic pair with actuator, $U$ : Universal joint) link mechanisms ${ }^{3}$. However, the 3-PUU TPM has been sensitive to alignment errors of the mechanism same as the $3-U \underline{P} U$ TPM ${ }^{4}$. The alignment error caused the undesirable tilt of the end plate of the TPM. By the tilt of the TPM, the standard deviation of the positioning error of the TMP along the $x$ and $y$ directions were very large ${ }^{5}$.

In this paper, a novel pure translational parallel mechanism is designed using a parallelogram link mechanism. The TPM is designed as positional sensitivities share an isotropic relationship at
the central position of the TPM, same as our previous prototype ${ }^{3}$. Moreover, the length of the rod is redesigned as to keep the isotropic characteristics over the working range of the TPM. The positional resolution of the TPM is designed under $1 \mu \mathrm{~m}$ in. 1.6 mm xyz cubic working range. In order to evaluate the accuracy of the TPM, an experimental system has been developed for the measurement of the 3D position of the probe ( $\phi 1.0 \mathrm{~mm}$ ) at the tip of the TPM using stereo cameras. Calibration of mechanical parts - link lengths, radii of the plates, and so on, - has been applied. The absolute accuracy of the TPM has become $3.4 \mu \mathrm{~m}$ in 1.6 mm xyz cubic working range.

## 2. Kinematics of TPM

### 2.1 Active Scanning Probe Using TPM

The TPM is attached to the tips of CMM or numerical-control machine tools and used as an active scanning probe as shown in Fig. 1 (a). The stylus is held by the TPM as mechanical impedances, mass, damping, and stiffness, - are spanned around the central (equilibrant) position of the TPM. The mechanical impedance is actively controlled by actuators equipped in the TPM. In general, the stiffness along the normal direction of the surface on the workpiece is set as small value. Active scan changes the mechanical impedance of the TPM in accordance with changing the normal direction of the curved surface. Positions of the stylus are continuously measured by positional sensors equipped in the TPM. Mass of the TPM's moving part is smaller than that of conventional scanning probe with the three stacked serial mechanism ${ }^{6,7}$ as illustrated in Fig. 1 (b). Small mass of
the TPM is advantageous to a quick response for scanning small zigzag shapes on the workpiece.

### 2.2 Design Concept

The schematic view of the TPM is shown in Fig. 2. In order to drive the end plate along the $x y z$ direction while keeping its orientation fixed, three legs $3-\underline{P} R_{1} R_{2} R_{2} R_{1}$ link mechanisms are employed. $\underline{P}$ denotes a prismatic pair driven by an actuator, $R$ denotes a rotational pair. The rotational pairs of the $\underline{P} R_{1} R_{2} R_{2} R_{1}$ link mechanism have limited two directions using a parallelogram linkage. Detail mechanism of the link will be described in section 4. All the linear actuators are fixed together at the base of the TPM. This arrangement is useful in obtaining a compact casing design for dust and drip proof at on-machine measurement. The link mechanisms are directory driven by small force controlled voice coil motors. Mechanical impedance control ${ }^{8}$ is installed for active scanning motion.

### 2.3 Kinematic Equations of TPM

Kinematic parameters of the TPM are illustrated in Fig. 3. The variables are defined as follows:
$\boldsymbol{p}:$ central position of the end plate; $\boldsymbol{p}=[x, y, z]^{T}$
$\boldsymbol{p}_{b i}$ : vector from the origin to $i$ th pair of the base plate
$\boldsymbol{p}_{e i}$ : vector from $\boldsymbol{p}$ to $i$ th pair of the end plate
$r_{b}$ : radius of the base plate, $r_{e}$ : radius of the end plate
$z_{i}$ : unit vector from pair of actuators $i$ to $i$ th pair of the end plate
$\boldsymbol{a}$ : unit vector from the actuator to pair of the end plate
$l_{i}$ : length of rod
$c_{i}$ : controlled value of $i$ th actuator
Our TPM is categorized in the parallel mechanism with vertically fixed linear actuators ${ }^{9}$. The relationship between the central position of the end plate and the controlled values of the actuators is expressed as

$$
\boldsymbol{L}_{i}=\left[\begin{array}{lll}
l_{x i} & l_{y i} & l_{z i} \tag{1}
\end{array}\right]^{T}=\boldsymbol{p}+\boldsymbol{p}_{e i}-\boldsymbol{p}_{b i}=c_{i} \boldsymbol{a}+l_{i} \boldsymbol{z}_{i}
$$

The differential of the kinematics is given as

$$
\begin{align*}
& \boldsymbol{J}_{1} \Delta \boldsymbol{p}=\boldsymbol{J}_{2} \Delta \boldsymbol{c} \\
& \Delta \boldsymbol{p}=[\Delta x, \Delta y, \Delta z]^{T}, \Delta \boldsymbol{c}=\left[\Delta c_{1}, \Delta c_{2}, \Delta c_{3}\right]^{T} .  \tag{2}\\
& \boldsymbol{J}_{1}=\left[\begin{array}{l}
z_{1}^{T} \\
z_{2}^{T} \\
z_{3}^{T}
\end{array}\right], \boldsymbol{J}_{1}=\left[\begin{array}{ccc}
z_{1}^{T} \boldsymbol{a} & 0 & 0 \\
0 & z_{2}^{T} \boldsymbol{a} & 0 \\
0 & 0 & z_{3}^{T} \boldsymbol{a}
\end{array}\right]
\end{align*}
$$

### 2.4 Inverse Kinematics

Inverse kinematics, essential for determining the actuator variables from the position of the end plate, is easily derived by using the quadratic equation ${ }^{9}$ from Eq. (1). By transporting $c_{i} \boldsymbol{a}$ of Eq. (1) to the left side, and multiplying each side by itself, we obtain

$$
\begin{equation*}
\left(\boldsymbol{L}_{i}-c_{i} \boldsymbol{a}\right)^{T} \cdot\left(\boldsymbol{L}_{i}-c_{i} \boldsymbol{a}\right)=\left(l_{i} z_{i}\right)^{T} \cdot\left(l_{i} \boldsymbol{z}_{i}\right) \tag{3}
\end{equation*}
$$

Inverse kinematics of the TPM is given by the analytical solution of Eq. (3) as

$$
\begin{equation*}
c_{i}=l_{z i}-\sqrt{l_{i}^{2}-l_{x i}^{2}-l_{y i}^{2}} \tag{4}
\end{equation*}
$$



Fig. 1 Parallel and serial mechanisms for active probing systems


Fig. 2 Schematic view of TPM


Fig. 3 Kinematic parameters of TPM

### 2.5 Direct Kinematics

Direction of the $i$ th pair of the base plate is denoted by angle $\varphi_{i}$, as shown in Fig. 3. Each pair of the base plate is assigned an equivalent angle as shown in Fig. 3. We similarly assign each pair of the end plate. In this case, $\boldsymbol{L}_{i}$ of Eq. (1) becomes as follows.

$$
\begin{align*}
& \boldsymbol{L}_{i}=\boldsymbol{p}-r_{b e} \boldsymbol{u}_{i} \\
& \boldsymbol{u}_{i}=\left[\begin{array}{lll}
\cos \varphi_{i} & \sin \varphi_{i} & 0
\end{array}\right]^{T}  \tag{5}\\
& r_{b e}=r_{b}-r_{e}
\end{align*}
$$

Equation (5) indicates that the kinematic parameter $\boldsymbol{L}_{i}$ is a function of the difference between the radii of the base plate $r_{b}$ and the end plate $r_{e}$. Hereafter, this difference in radii will be denoted as $r_{b e}$. By applying Eq. (5) to Eq. (3), we obtain

$$
\begin{align*}
& \left(x-r_{b e} \cos \varphi_{i}\right)^{2}+\left(y-r_{b e} \cos \varphi_{i}\right)^{2}+\left(z-c_{i}\right)^{2}=l_{i}^{2}  \tag{6}\\
& (i=1, . .3)
\end{align*}
$$

Equation (6) indicates three spheres in the space of the end plate position $\boldsymbol{p}=[x, y, z]^{T}$. Direct kinematics, essential for determining the position of the end plate from the actuator variables, is obtained as the intersection of these spheres.

## 3. Optimum Design of TPM

### 3.1 Manipulability Ellipsoid

Equation (2) is evaluated using the manipulability ellipsoid ${ }^{10}$. Small motions of the actuators $\Delta \boldsymbol{c}$ are transferred to small motions of the end plate $\Delta \boldsymbol{p}$ as

$$
\begin{equation*}
\Delta \boldsymbol{p}=\left(\boldsymbol{J}_{1}^{-1} \boldsymbol{J}_{2}\right) \Delta \boldsymbol{c}=\boldsymbol{J}_{12} \Delta \boldsymbol{c} \tag{7}
\end{equation*}
$$

The TPM is controlled such that the scanning motion is executed around the central position of the end plate, i.e., $x=y=0$. The singular value decomposition of $\boldsymbol{J}_{12}$ at the central position is derived using symbolic mathematical manipulation as follows.

$$
\begin{align*}
& \boldsymbol{J}_{12}=\boldsymbol{U} \boldsymbol{\Sigma} \boldsymbol{V}^{T} \\
& \boldsymbol{U}=\boldsymbol{I}, \boldsymbol{\Sigma}=\operatorname{diag}\left(\sigma_{x}, \sigma_{y}, \sigma_{z}\right)  \tag{8}\\
& \sigma_{x}=\sigma_{y}=\sqrt{2 / 3} \cdot \sqrt{\left(l_{i} / r_{b e}\right)^{2}-1}, \sigma_{z}=\sqrt{1 / 3}
\end{align*}
$$

At the central position, the link angle $\gamma$ in Fig. 2 is given as

$$
\begin{equation*}
\gamma=\tan ^{-1} \sqrt{\left(l_{i} / r_{b e}\right)^{2}-1} \tag{9}
\end{equation*}
$$

### 3.2 Isotropic Design

The optimum design of the TPM is defined as that in which the manipulability of the mechanism is equal in all directions. This implies that the sensitivities of the forces and positions of the mechanism become isotropic. The optimum condition is realized when the relationship between $l_{i}$ and $r_{b e}$ is given as

$$
\begin{equation*}
r_{b e} / l_{i}=\sqrt{2 / 3} \tag{10}
\end{equation*}
$$

Under this condition, the link angle $\gamma$ yields

$$
\begin{equation*}
\gamma=\tan ^{-1} \sqrt{1 / 2}=35.26(\mathrm{deg}) \tag{11}
\end{equation*}
$$

## 4. Design and Prototype of TPM

### 4.1 Pure Translational Motion

We newly design a pure translational parallel mechanism using a parallelogram linkage as shown in Fig. 4. The parallelogram linkage has four rotational pairs, whose rotational axes are $\boldsymbol{t}_{1}, \boldsymbol{t}_{2 a}, \boldsymbol{t}_{2 b}$ and $\boldsymbol{t}_{3}$. As shown in Fig. 4 (b) and (c), by mechanical constraints of the parallelogram linkage, $\boldsymbol{t}_{1}$ and $\boldsymbol{t}_{2 a}$ are parallel to $\boldsymbol{t}_{3}$ and $\boldsymbol{t}_{2 b}$, respectively, i.e.,

$$
\begin{equation*}
\boldsymbol{t}_{1} / / \boldsymbol{t}_{3}, \boldsymbol{t}_{2 a} / / \boldsymbol{t}_{2 b} \tag{12}
\end{equation*}
$$

This type of link mechanism is categorized into three legs 3$\underline{P} R_{1} R_{2} R_{2} R_{1}$ mechanisms. The rotational pairs of the $\underline{P} R_{1} R_{2} R_{2} R_{1}$ link mechanism have limited two rotational directions. Recently, type synthesis of parallel mechanism with limited degrees of freedom has been summarized by Kong and Gosselin ${ }^{11}$. The $3-\underline{P} R_{1} R_{2} R_{2} R_{1}$ mechanism belongs to the type of pure translational motion.

### 4.2 Design of Joint Mechanism

Cross-section around the end plate joint of the newly designed
parallelogram link is shown in Fig. 5. Three pairs of miniature ball bearings are built in the joint. Inner diameter of the bearing is 1.0 mm . Small screws are machined at the end of the shaft 1 and 2 . The shafts and nuts pressurize the miniature bearings of axes $t_{2 b}$. The screw 1 and 2 pressurize the miniature bearings of axes $\boldsymbol{t}_{3}$. By controlling the fasten torques of these screws, appropriate amount of pressures are applied to the bearings as removing backlashes but avoiding excessive frictions. Tolerance between the outer ring of the bearing and the casing, and that between the inner ring and the shaft are designed such that they closely fit each other.

### 4.3 Design of Rod Length and Radii of Plates

Moving coil motor, LAL-10 by SMAC Inc, with stroke length of $\pm 2.5 \mathrm{~mm}$, rated force of 1.3 N , and positional resolution of $1 \mu \mathrm{~m}$, is used for linear actuator of the prototype. We designed the rod length $l_{i}$ of the previous prototype as $8.1 \mathrm{~mm}^{3}$. However, shorter length of rod degrades the isotropy of the mechanism when the TPM is displaced far from the central position. Here, an evaluation scale for isotropy $s_{t}$ is described using the condition number of matrix $\boldsymbol{J}_{12}$ in Eq. (7) as

$$
\begin{equation*}
s_{t}(x, y)=1 / \operatorname{cond}\left(\boldsymbol{J}_{12}\right)=\sigma_{\min } / \sigma_{\max } \leq 1 \tag{13}
\end{equation*}
$$

In case of our TPM, that the direction $\boldsymbol{a}$ of each actuator coincides with the $z$ axis, matrix $J_{12}$ is independent from the member $z$ of the end plate position $\boldsymbol{p} . s_{t}$ is function of members $x$ and $y$ of $\boldsymbol{p}$. At the central position of the TPM with optimal design, the isotropy $s_{t}$ becomes 1 . The isotropy is getting smaller when the TPM is displaced farther from the central position.

In consideration of the stroke length of the linear actuator, nominal working range of the end plate position $\boldsymbol{p}=[x, y, z]^{T}$ is designed as

$$
\begin{equation*}
-0.8 \mathrm{~mm} \leq x, y, z \leq+0.8 \mathrm{~mm} \tag{14}
\end{equation*}
$$



Fig. 4 Parallelogram linkage


Fig. 5 Cross-section view of end plate joint

Relation between the rod length $l_{i}$ and minimum isotropy $s_{i}$ in the nominal working range is shown in Fig. 6. The isotropy is exponentially decreasing when the rod length is designed smaller than 15 mm . The rod length of the prototype is designed as 18.0 mm for compactness of the mechanism. In order to keep the space for the force sensor, radius of the end plate $r_{e}$ is designed as 15.47 mm . Radius of the base plate $r_{b}$ is designed as 30.17 mm by the optimum condition of Eq. (10).

Actual working space of the TPM with above design is shown in Fig. 7 (a). The nominal working range of Eq. (14) is superimposed as a cube in the figure. The isotropy of the TPM with changing $x$ and $y$ of end plate position $\boldsymbol{p}$ is shown in Fig. 7 (b). At the central position of the TPM, i.e. $x=y=0$, the isotropy $s_{t}$ becomes 1 . The isotropy is decreased at the border of the working range. The isotropy keeps $76 \%$ of fully isotropic condition $\left(s_{t}=1\right)$ even though the TPM moves to the border of the working range.

### 4.4 Prototyping TPM

Prototype of the TPM is shown in Fig. 8. Three axes force sensor, MX032 by Minebea Co., with rated force of $\pm 2.5 \mathrm{~N}$ is mounted on the end plate. A ruby stylus by Renishaw with ball radius of 0.5 mm is attached on the force sensor.

## 5. Calibration of Mechanism

### 5.1 Experimental Setup

In order to evaluate the accuracy of the prototype, 3D measurement system using parallel lighting stereo cameras is designed as shown in Fig. 9. Parallel back light projects shadow image of the stylus on to the $1 / 2$ inch CCD camera, CV-A1 by JAI with resolution of $1380 \times 1035$ pixels. Telecentric lens, TS SILVER by Edmund Optics Inc. with magnification of $\times 1$, is mounted on the camera. Image range of the camera with the lens becomes 6.4 mm $\times 4.8 \mathrm{~mm}$. One pixel of image corresponds to $4.6 \mu \mathrm{~m} \times 4.6 \mu \mathrm{~m}$ in size. The central position of the ruby ball on the camera plane is calculated using positions of the edge on gray image of the projected circle. The positions on the edge are measured by sub-pixel image procession. The 3D position of the ruby ball is calculated using central positions of projected image and positional relations of the stereo cameras.

The prototype is surmounted on XYZ stage as shown in Fig. 9. Positional resolution of the stage is $1 \mu \mathrm{~m}$. The stylus is positioned as cubic grid with edge length of 2.0 mm and spacing of 0.5 mm using the stage while motion of the prototype is fixed. Position of each lattice point of the cubic grid is measured by the camera as the central position of the ruby ball. The positional relations of the stereo cameras are calibrated using the data. The relative resolution of the system is $1.0 \mu \mathrm{~m}$. The absolute accuracy of the 3 D measurement system after calibration has become $3.4 \mu \mathrm{~m}$.

### 5.2 Error Propagation

If actual values of the mechanical parameters such as length of the rod $l_{i}$, radius of the base plate $r_{b}$, or radius of the end plate $r_{e}$ differs from the designed ones, the central position of the end plate $\boldsymbol{p}$ differs from the desired position. The error propagation analysis is applied to investigate the influence from the errors of mechanical parameter to the error of the position $\boldsymbol{p}$. In this paper, error analysis and kinematical parameter calibration are applied using the kinematic
model of Eq. (1). In fact there still exist other errors such that lengths of corresponding links or angles of corresponding axes in the parallelogram mechanism are not identical. Some of these errors may cause not only collapse condition of the pure translational motion but also deformations of the mechanism. These errors can not be modeled by the kinematic model of Eq. (1). Modeling of these errors is our future problem.


Fig. 6 Relationship between the rod length and isotropy of TPM

(a) Operating range

(b) Isotropy

Fig. 7 Operating range and Isotropy of TPM


Fig. 8 Prototype of TPM


Fig. 9 Experimental system for measuring the position of TPM

Table 1 Error propagation at the central position of the TPM

|  | $\Delta c_{1}$ | $\Delta c_{2}$ | $\Delta c_{3}$ | $\Delta l_{1}$ | $\Delta l_{2}$ | $\Delta l_{3}$ | $\Delta r_{b e}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta x$ | 0.471 | -0.236 | -0.236 | 0.816 | -0.408 | -0.408 | 0.000 |
| $\Delta y$ | 0.000 | 0.408 | -0.408 | 0.000 | 0.707 | -0.707 | 0.000 |
| $\Delta z$ | 0.333 | 0.333 | 0.333 | 0.578 | 0.578 | 0.578 | -1.415 |
| $d$ | 0.577 | 0.577 | 0.577 | 1.000 | 1.000 | 1.000 | 1.415 |

By taking the total differential of Eq. (1) after substituting Eq. (6) to Eq. (1), we obtain

$$
\begin{equation*}
\Delta \boldsymbol{p}-\Delta r_{b e} \boldsymbol{u}_{e i}-r_{b e} \Delta \boldsymbol{u}_{e i}=\Delta c_{i} \boldsymbol{a}+c_{i} \Delta \boldsymbol{a}+\Delta l_{i} z_{i}+l_{i} \Delta z_{i} . \tag{15}
\end{equation*}
$$

For simplicity, $\boldsymbol{u}_{e i}$ and $\boldsymbol{a}$ in Eq. (15) are assumed to be constant. In this paper, three positions of the linear actuators $\left(c_{1}, c_{2}\right.$, and $\left.c_{3}\right)$, three lengths of the rods $\left(l_{1}, l_{2}\right.$, and $\left.l_{3}\right)$, and the difference of radii of the base and the end plate $\left(r_{b e}\right)$, are set as targets of error analysis and parameter calibration. These parameters do not influence on the structure of the TPM. Even if the values of these parameters differ from the designed ones, the TPM keeps the condition of the pure translational motion and causes no deformations of the mechanism.

By multiplying $z_{i}^{T}$ to the both sides of Eq. (15), and using relations such as $z_{i}^{T} z_{i}=1$ and $z_{i}^{T} \Delta z_{i}=0$, we obtain

$$
\begin{equation*}
z_{i}^{T} \Delta \boldsymbol{p}=\boldsymbol{z}_{i}^{T} \boldsymbol{a} \Delta c_{i}+\Delta l_{i}+z_{i}^{T} \boldsymbol{u}_{e i} \Delta r_{b e} \tag{16}
\end{equation*}
$$

Errors of the kinematic parameters propagate to the positioning error of the TPM via the error propagation matrix $\boldsymbol{J}_{e}$ as follows.

$$
\begin{align*}
\Delta \boldsymbol{p} & =\boldsymbol{J}_{e} \Delta \boldsymbol{q} \\
\boldsymbol{J}_{e} & =\left[\begin{array}{l}
z_{1}{ }^{T} \\
z_{2}{ }^{T} \\
z_{3}{ }^{T}
\end{array}\right]^{-1} \cdot\left[\begin{array}{ccccccc}
z_{1}{ }^{T} \boldsymbol{a} & 0 & 0 & 1 & 0 & 0 & z_{1}{ }^{T} \boldsymbol{u}_{1} \\
0 & z_{2}{ }^{T} \boldsymbol{a} & 0 & 0 & 1 & 0 & z_{2}{ }^{T} \boldsymbol{u}_{2} \\
0 & 0 & z_{3}{ }^{T} \boldsymbol{a} & 0 & 0 & 1 & \boldsymbol{z}_{3}{ }^{T} \boldsymbol{u}_{3}
\end{array}\right]  \tag{17}\\
\Delta \boldsymbol{q} & =\left[\Delta c_{1}, \Delta c_{2}, \Delta c_{3}, \Delta l_{1}, \Delta l_{2}, \Delta l_{3}, \Delta r_{b e}\right]^{T}
\end{align*}
$$

Quantity of each element of the error propagation matrix $J_{e}$ expresses a degree of error propagation influence of the corresponding parameter. For example, quantity of the $1^{\text {st }}$ row and $4^{\text {th }}$ column of the $J_{e}$ represents the quantity of influence from $\Delta l_{1}$ to $\Delta x$. Values of $\boldsymbol{J}_{e}$ at the central position of the TPM are shown as Table 1. The bottom row of the Table 1 shows distance $d$ between the desired position and actual position with errors of the kinematic parameters as

$$
\begin{equation*}
d=\sqrt{(\Delta x)^{2}+(\Delta y)^{2}+(\Delta z)^{2}} \tag{18}
\end{equation*}
$$

If the actual rod length $l_{1}$ differs 0.1 mm from the designed one, positioning errors of the TPM are estimated by the Table 1 as follows.

$$
\begin{align*}
\Delta x & =0.816 \times 0.1 \mathrm{~mm}=0.082 \mathrm{~mm} \\
\Delta y & =0.000 \times 0.1 \mathrm{~mm}=0.0 \mathrm{~mm}  \tag{19}\\
\Delta z & =0.578 \times 0.1 \mathrm{~mm}=0.058 \mathrm{~mm} \\
d & =1.000 \times 0.1 \mathrm{~mm}=0.1 \mathrm{~mm}
\end{align*}
$$

Positioning errors of the TMP by the kinematic errors are not negligible. In the following section, actual values of the kinematic parameters are estimated for improving the accuracy of the TPM.

### 5.3 Calibration Algorithm

Calibration method with the direct kinematics (DK method) based on Eq. (6) is applied. The DK method is one of the typical methods
for parameter calibration of parallel mechanisms ${ }^{12}$. Several set of actuator positions, $\boldsymbol{c}_{j}(j=1, \ldots, n)$, are given to the actual TPM and the direct kinematic model of the TPM. End plate positions of the actual TPM, $\boldsymbol{x}_{m j}(j=1, \ldots, n)$, are measured by external sensors. Positions of the mathematical model of the TPM, $\boldsymbol{x}_{j}(j=1, \ldots, n)$, are calculated using the direct kinematics. Kinematic parameters are estimated as the positions of the corresponding $\boldsymbol{x}_{m j}$ and $\boldsymbol{x}_{j}$ are close in value using the least square method. In addition, we append positions and orientations of the base coordinate frame of the TPM with respect to the world (camera) coordinate frame to the calibration targets.

Coordinate transformation matrix ${ }^{w} \boldsymbol{A}_{p}$ form the world coordinate frame $\Sigma_{w}$ to the base coordinate frame of the TPM $\Sigma_{p}$ is defined as

$$
\begin{align*}
& { }^{w} \boldsymbol{A}_{p}\left(\boldsymbol{q}_{c}\right)=\boldsymbol{\operatorname { T r a n s }}\left(x_{0}, y_{0}, z_{0}\right) \cdot \boldsymbol{\operatorname { R o t }}\left(z, \gamma_{0}\right) \cdot \boldsymbol{\operatorname { R o t }}\left(y, \beta_{0}\right) \cdot \boldsymbol{\operatorname { R o t }}\left(x, \alpha_{0}\right) \\
& \boldsymbol{q}_{c}=\left[x_{0}, y_{0}, z_{0}, \alpha_{0}, \beta_{0}, \gamma_{0}\right]^{T} . \tag{20}
\end{align*}
$$

In Eq. (20), Trans and Rot are $4 \times 4$ translational and rotational matrices ${ }^{13}$, respectively. In convenient, parameters of the coordinate transformation are packed in vector $\boldsymbol{q}_{c}$. The direct kinematics of the TPM is given as a solution of Eq. (6).

$$
\begin{equation*}
{ }^{p} \boldsymbol{x}_{j}=\left[{ }^{p} x_{j},{ }^{p} y_{j},{ }^{p} z_{j}, 1\right]^{T}={ }^{p} \boldsymbol{x}_{j}\left(\boldsymbol{q}_{l}, \boldsymbol{c}_{j}\right) \tag{21}
\end{equation*}
$$

The left superscript " $p$ " indicates that position $\boldsymbol{x}_{j}$ is defined with respect to the frame $\Sigma_{p}$. Kinematic parameters for calibration are packed in vector $\boldsymbol{q}_{l}$ as

$$
\begin{equation*}
\boldsymbol{q}_{l}=\left[c_{20}, c_{30}, l_{1}, l_{2}, l_{3}, r_{b e}\right]^{T} \tag{22}
\end{equation*}
$$

In Eq. (22), $c_{k 0}$ represents the encoder offset, distance between the mechanical origin of the TPM and the origin of the positional sensor (linear encoder) of $k^{\text {th }}$ actuator. The coordinate frames and the kinematic parameters concerning to the calibration are illustrated in Fig. 10 (a). Actuator positions $\boldsymbol{c}_{j}$ are given that the TPM is positioned as cubic grid with edge length of 1.6 mm and spacing of 0.4 mm as shown in Fig. $10(\mathrm{~b})$. The number of data is $125(=n)$. Each position of the TMP with respect to the world frame, ${ }^{w} \boldsymbol{x}_{m j}$, is measured by the stereo camera as the central position of the ruby ball. The kinematic parameters $\boldsymbol{q}_{l}$ and the coordinate parameters $\boldsymbol{q}_{c}$ are estimated by the nonlinear least square method as minimizing the following cost function.

$$
\begin{align*}
& \sum_{j=1}^{n} d_{j}=\sum_{j=1}^{n}\left\|{ }^{w} \boldsymbol{x}_{m j}-{ }^{p} \boldsymbol{x}_{j}\left(\boldsymbol{q}_{c}, \boldsymbol{q}_{l}, \boldsymbol{c}_{j}\right)\right\|  \tag{23}\\
& { }^{p} \boldsymbol{x}_{j}\left(\boldsymbol{q}_{c}, \boldsymbol{q}_{l}, \boldsymbol{c}_{j}\right)={ }^{w} \boldsymbol{A}_{p}\left(\boldsymbol{q}_{c}\right)^{p} \boldsymbol{x}_{j}\left(\boldsymbol{q}_{l}, \boldsymbol{c}_{j}\right)
\end{align*}
$$


(a) Calibration model

(b) Data set for calibration

Fig. 10 Model and data set for calibration of TPM


Fig. 11 Calibration Result of case 1 and case 4
Table 2 Result of the calibration

|  |  | case 1 | case 2 | case 3 | case 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{q}_{c}$ | $x_{0}(\mathrm{~mm})$ | 0.070 | 0.071 | 0.173 | 0.169 |
|  | $y_{0}(\mathrm{~mm})$ | 0.088 | 0.089 | -0.042 | -0.041 |
|  | $z_{0}(\mathrm{~mm})$ | -9.999 | -10.040 | -10.137 | -9.933 |
|  | $\alpha_{0}(\mathrm{deg})$ | 0.488 | 0.126 | 0.371 | 0.371 |
|  | $\beta_{0}(\mathrm{deg})$ | -0.413 | -0.254 | -0.063 | -0.064 |
|  | $\gamma_{0}(\mathrm{deg})$ | 0.152 | 0.151 | 0.161 | 0.161 |
| $\boldsymbol{q}_{l}$ | $c_{02}(\mathrm{~mm})$ | [0.000] | 0.018 | 0.269 | 0.264 |
|  | $c_{03}(\mathrm{~mm})$ | [0.000] | -0.141 | -0.113 | -0.110 |
|  | $l_{1}(\mathrm{~mm})$ | [18.000] | [18.000] | 17.948 | 17.597 |
|  | $l_{2}(\mathrm{~mm})$ | [18.000] | [18.000] | 18.380 | 18.020 |
|  | $l_{3}(\mathrm{~mm})$ | [18.000] | [18.000] | 18.005 | 17.653 |
|  | $r_{\text {be }}(\mathrm{mm})$ | [14.700] | [14.700] | [14.700] | 14.411 |
|  | $d_{\text {mean }}(\mu \mathrm{m})$ | 16.2 | 15.2 | 3.5 | 3.4 |

### 5.4 Results and Discussions

The targets of the calibration are set as following four cases.
(case 1): $\boldsymbol{q}_{c}$ (coordinate parameters)
(case 2): (case 1 ) $+c_{20}$ and $c_{30}$ (encoder offset)
(case 3): (case 2$)+l_{1}, l_{2}$ and $l_{3}$ (length of rod)
(case 4): (case 3$)+r_{b e}$ (difference between end and base radii)
Results of the calibrations are shown in Table 2. Absolute accuracy of the calibration is evaluated using $d_{\text {mean }}$, the average of distances $d_{j}$ in Eq. (23)

Figure 11 shows the measurement position $\boldsymbol{x}_{m j}$ and the model position $\boldsymbol{x}_{j}$ after calibration about case 1 and case 4 . White circles represent the position of $\boldsymbol{x}_{m j}$. Error vector $\boldsymbol{e}_{j}=\boldsymbol{x}_{j}-\boldsymbol{x}_{m j}$ is superimposed in Fig. 11 as solid line. Each length of the error vector is scaled up 25 times. As shown in Fig. 11 (a), there exist not random but systematic (biased) errors attributed by errors of kinematic parameters of the TPM. As shown in Table 2 and Fig. 11 (b), the absolute accuracy of the TPM is greatly improved by the kinematic parameter calibration. In the experiment, the absolute accuracy of the TPM has become $3.4 \mu \mathrm{~m}$ in $1.6 \mathrm{~mm} x y z$ cubic working range. Improving the accuracy of the stereo cameras and modifying the kinematic model of the TPM are our future problems for obtaining further accuracy of the TPM.

## 6. Conclusion

A 3-dof active scanning probe using a translational parallel mechanism (TPM) is proposed. The characteristics of the mechanism, optimum design, and accuracy of the TPM have been introduced.
(1) Optimum TPM design has been proposed as that the positional measuring sensitivities share isotropic relationships.
(2) Prototype of the TPM using a parallelogram linkage has been
developed. It achieves pure translational parallel mechanism. The working range and the positional resolution of the mechanism have been designed as $\pm 0.8 \mathrm{~mm}$ cubic volume and $1 \mu \mathrm{~m}$, respectively.
(3) In order to evaluate the accuracy of the TPM, an experimental system for the measurement of the position of the TPM by using image processing has been developed.
(4) By applying mechanical calibration, the accuracy of the absolute position of the TPM has become $3.4 \mu \mathrm{~m}$.

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