General method for phase shifting shadow moiré by iterative least-squares fitting

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Abstract

The method of phase-shifting shadow moiré is studied and analyzed. Based on this, a general flexible method for phase shifting shadow moiré by iterative least-squares fitting is proposed. In our proposed system, the grating is translated in arbitrary distance to introduce phase shifts. In order to minimize the effects of the phase-shifting errors caused by dependent effects, the sampled interferogram is divided into many blocks and the phase value at each point of every block is determined by the Kong algorithm, which base on arbitrary phase- shifting. In the end, simulation and experimental results prove the feasibility of the method.

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1. Introduction

The shadow moiré technique [1] has been well known as an optical noncontact method for mapping the 3-D shape of objects for its simple setup with no special requirements for light coherence, mechanical isolation, etc [2]. Since 1988, by combining temporal phase shifting method with shadow moiré technique, various procedures have been proposed to increase its sensitivity [3-7].But, in all these studies, they else assumed the intensity distribution of shadow moiré fringes has a linear dependence on the height distribution of the object to be measured[3-5] or adopted a complicated setup[6-7]. In fact, due to the nonlinear nature of the height-phase relation, uniform phase shifts across the field of view of shadow moiré are impossible [4]. Naturally, the application of the global phase shifting method to shadow moiré can only be fulfilled under certain approximations [8]. The price to pay is the limited measurable depth range. In this sense, the application of the constant but unknown relative step phase shifting algorithm to shadow moiré may be a good choice to overcome the error caused by height dependent effects. However, this kind of algorithm is insensitive to miscalibration of the phase shifter. On the other hand, in practice, due to real, imperfect interferometric measurements, such as improper phase shifts, nonlinear photodetector response, observed waveforms such as fringes in an interferogram often become non-sinusoidal [9]. In this case, using Carré algorithm [10] will fail to fulfill the task of measuring.

In order to overcome the error caused by both height dependent effects and miscalibration of the phase shifter. In this paper, we divide the interferogram space into several blocks (e.g. 16×16), if the blocks are sufficiently small, we may consider that the phase shifts in each block do not have pixel-to-pixel variation and can be assumed as constants. Then, the well known Kong algorithm [11] is applied into each block to attract the measurement phase. In the end, simulation and experimental results prove the feasibility of the method.

2. Principle

2.1 Methods

In Fig. 1 we show the experimental shadow moiré setup employed. It is composed by: (1) a Ronchi grating with a pitch p, which is placed in the x y plane and close to the surface to be measured, (2) a camera and (3) a light source are placed at distance h from the grating. And d is the distance between the light source and the camera. When the light source illuminates the grating on the specimen, in theory, the measured intensity $I(x, y)$ at the grating plane recorded by the observer can be described by

$$I^r(x,y) = A(x,y) + B(x,y)\cos[\phi(x,y)] \qquad (1)$$

Where $A(x, y)$ the intensity is bias, and $B(x, y)$ is modulation factor. The phase $\phi(x, y)$ is described by the following expression:

$$\phi(x,y) = 2\pi dz(x,y)/p[h+z(x,y] \qquad (2)$$

Where $z(x, y)$ represents the distance from the grating plane to a point (x, y) on the object. Suppose the grating in Fig. 1 is shifted vertically in the direction perpendicular to the grating m times. Thus generates m positions. In each position, the grating has an arbitrary distance Δh_i $(i = 0,1,......,m)$ with respect to the original grating position and the parameters need not be carefully calibrated. The generated phase shift can be expressed as

$$\delta_i(x,y) = 2\pi d\Delta h_i / p[h+z(x,y)] \qquad (3)$$

It is clear that the generated phase shift is non-uniform across the field of view. Thus, the intensity of the m interferograms with unknown phase shift can be described as

$$I^r{}_i(x,y) = A(x,y) + B(x,y)\cos[\phi(x,y) + \delta_i(x,y)] \qquad (4)$$

Eq. (3) can be rewritten as

$$\delta_i(x,y) = \zeta_i + \varepsilon_i + \gamma_i(x,y) \qquad (5)$$

Where $\zeta_i = 2\pi d\Delta h_i / ph$ denotes the estimate of the $\delta_i(x, y)$ in the condition of $h >> z(x, y)$, which is principal component of the $\delta_i(x, y)$, ε_i represent constant phase-shifting errors caused by miscalibration of the grating translation, and $\gamma_i(x, y)$ is a phase-shifting error caused by ignoring height dependent effects. In traditional phase-shifting shadow moiré system, $\gamma_i(x, y)$ is always neglected, so error arise.

In order to compensate the measurement error caused by $\gamma_i(x, y)$, here we divide the interferogram space into several blocks (e.g. 16×16), if the blocks are sufficiently small, we may consider that the phase shifts in each block do not have pixel-to-pixel variation and can be assumed as constants. Then we can apply the well known Kong algorithm into each block to extract the measurement phase, which based on arbitrary phase shift, can take the actual values of phase shifts as unknowns together with the wavefront phases ultimately to be determined. So, the precision phase can be retrieved from one block after another. So, in the next section, our discussion is limited in one block of each frame of the interferogram.

2.2 Determination of the Phase

In one block of ith frame of the interferogram, Defining a new set of variables as $a = A, b = B\cos\phi, c = -B\sin\phi$, and $I^r{}_i$ is substituted by the experimentally measured intensity of the interferogram $I^m{}_i$, where the spatial dependence (x, y) of functions I^r, A, B, ϕ has been dropped for simplicity. So we can rewriting Eq. (4) in the form of matrix equation as

$$JX = I^m \qquad (6)$$

where,

$$J = \begin{bmatrix} 1 & 1 & 0 \\ 1 & \cos\delta_1 & \sin\delta_1 \\ ...... & ...... & ...... \\ 1 & \cos\delta_m & \sin\delta_m \end{bmatrix}, \quad X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \quad I^m = \begin{bmatrix} I^m{}_0 \\ I^m{}_1 \\ ...... \\ I^m{}_m \end{bmatrix}$$

When δ_i is substituted by ζ_i, then we get the least squares solution X^* by

$$J^T JX^* = J^T I \qquad (7)$$

When the calculation results are used, the phase value is given by

$$\phi = a\tan 2(-c, b) \qquad (8)$$

2.3. Determination of the phase shift

With the result solved by Sec. B, we obtain

$$(I^r{}_i - A)/B = \cos[\phi + i\delta] \qquad (9)$$

And

$$B = \sqrt{b^2 + c^2} \qquad (10)$$

Generally speaking, the phase-shifting in shadow moiré varies with the pixel, but in one block, the variation of height is so small that the phase shifts in each block can be assumed as constants psh_i. Then the phases in the block solved by Eq. (8) are denoted as $\phi_1, \phi_2, ... \phi_m$ (m is the total number of pixel in the block). Defining a new set of variables as $I_i' = (I^r{}_i - A)/B, b_i' = \cos\delta_{b_i}, c_i' = -\sin\delta_{bi}$, and $I^r{}_i$ is substituted by I_{mi}, we have

$$J' A_i' = I' \qquad (11)$$

where,

$$J' = \begin{bmatrix} \cos\phi_1 & \sin\phi_1 \\ \cos\phi_2 & \sin\phi_2 \\ ...... & ...... \\ \cos\phi_m & \sin\phi_m \end{bmatrix}, \quad A_i' = \begin{bmatrix} b_i' \\ c_i' \end{bmatrix}, \quad I' = \begin{bmatrix} I_0 \\ I_1 \\ ...... \\ I_m \end{bmatrix}$$

Using the method above, the phase shift are gotten by

$$psh_i = a\tan 2(-c_i', b_i') \qquad (12)$$

Substituting Eq. (12) into Eq. (6), so, we can repeat the procedure above alternately until numerical convergence and the required accuracy are achieved. The convergence criterion for relative phase shifts can be expressed as:

$$\max(|\delta_i^q - \delta_i^{q-1}|) < \varepsilon \qquad (13)$$

Where q represents the number of iterations and ε is a predefined accuracy requirement. When the convergence criterion is satisfied, the correct height distributions are determined by

$$z^u = \frac{ph\phi}{2\pi d - p\phi} \qquad (14)$$

3. Simulation and Experiments

Since the exact expression of an actual object surface is hard to know due to many practical factors, a series of computer simulations have been carried out to verify the effectiveness of the proposed algorithm. To test the proposed algorithm accuracy, we define the actual height map as $z = 0.2x$ mm, the parameters for simulation as $p = 0.05mm, d = 100mm, h = 160mm, \Delta h = 0.02mm$, and according to Eq. (4), the intensity distributions as $I^r{}_i = 127 + 128\cos(\phi + i\delta)$, where $0 \le x \le 2.55, 0 \le y \le 2.55$, and the total number of pixels in the x, y directions both equal 256. By changing the i value a set of five phase-shifted interferograms is obtained. According to Eq. (5), if we neglect the effect of phase shift error, then the phase shift will be $0, \pi/2, \pi, 3\pi/2, 2\pi$. In order to illustrate the performance of the proposed iterative algorithm the random error of grating translation is also added into the five phase-shifted interferograms and the real distance of grating translation is defined as $\Delta h = 0.02(1 + 0.01 * rand())$, where $rand()$ is a Matlab function which returns a pseudorandom numbers.

Then the heights are extracted by the proposed iterative algorithm, Bi algorithm [12], Schwider-Hariharan algorithm, and Kong algorithm. The error function defines as $e(x, y) = abs(z(x, y) - h(x, y))$, where $h(x, y)$ is demodulation results. Figure 2 shows the residual errors $e(x, 0.9)$ solved by the four algorithms. It is evident that the proposed iterative algorithm can further reduce the fluctuation measurement errors in the presence of uniform phase-shifting error and miscalibration phase-shifting error.

To further verify the proposed method, a gauge block, which is tilted for a small degree, is selected as the specimen. The experimental setup used in the verification is illustrated in Figs. 1. The red LED is used as light source and the CCD is selected as detector,
where
$p = 0.05mm, d = 100mm, h = 160mm, \Delta h = 0.02mm$.

During our experiment, the grating is shifted by micrometer stage vertically for 4 times. Then 5 phase-shifted interferograms are recorded. Then, the proposed algorithm is applied to retrieve the 3-D shape of the object. Fig. 3 shows the demodulation results. Compared with the specimen, the proposed phase-shifting shadow moiré technique successfully retrieved the 3-D shape of the object.

4. Conclusions

To conclude, we have proposed a new algorithm for phase shifting shadow moiré to compensate the measurement error caused by both uniform phase-shifting error and miscalibration phase-shifting error. The determination of phase in the proposed algorithm is based on the iterative least-squares fitting. Experiments and simulation demonstrate the feasibility of the proposed algorithm. The further study will be focused on the application of this method into industrial automatic dimensional inspection.

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Fig.1 Optical arrangement of shadow moiré

Fig.2. Measurement errors in the presence of uniform and miscalibration phase-shifting errors

(a) Bi’s algorithm. (b) Schwider-Hariharan algorithm. (c) Proposed algorithm. (d) Kong’s algorithm.

Fig. 3. Results of demodulation using the first five images.

Fig.1

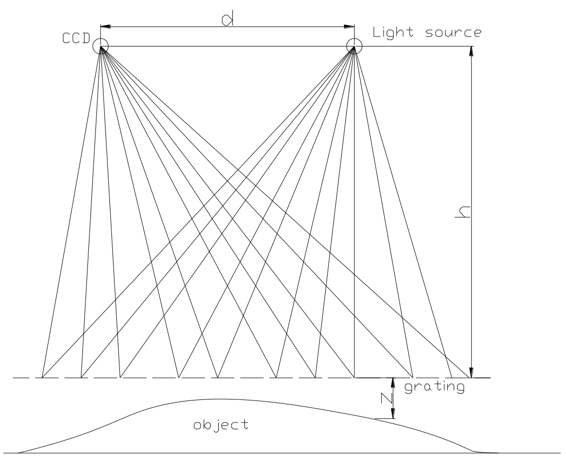
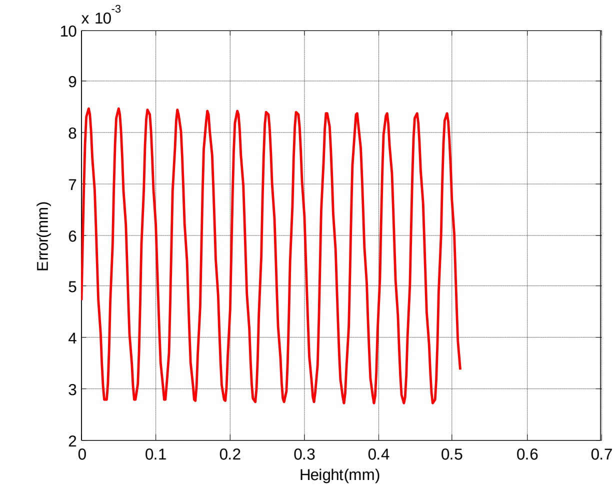
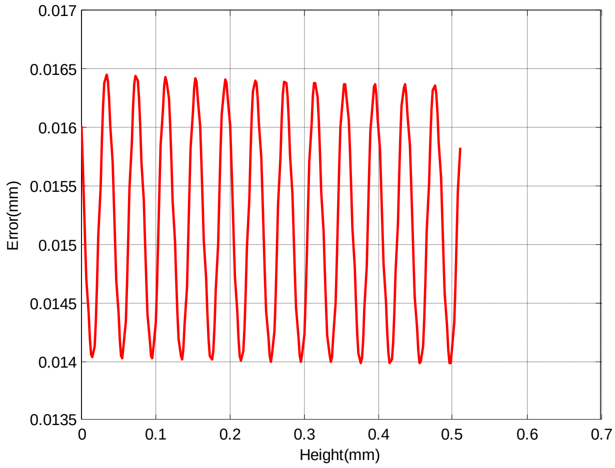


Fig.2



(a)



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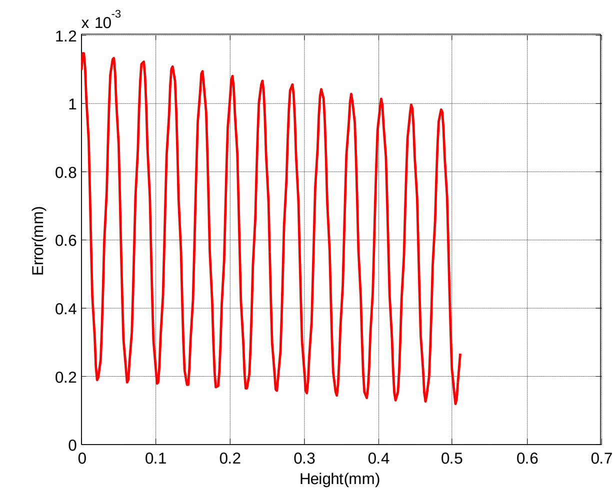

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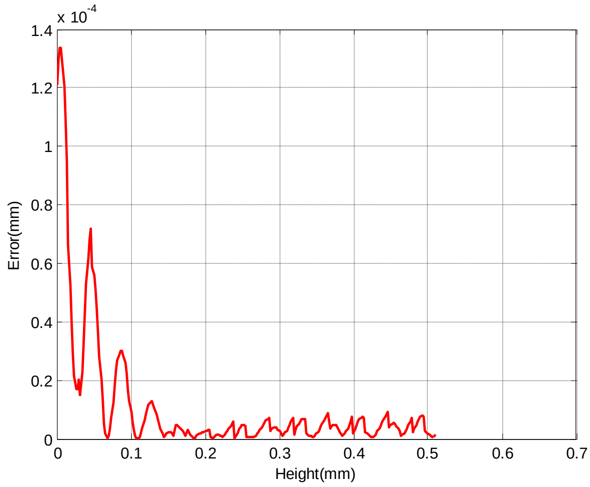
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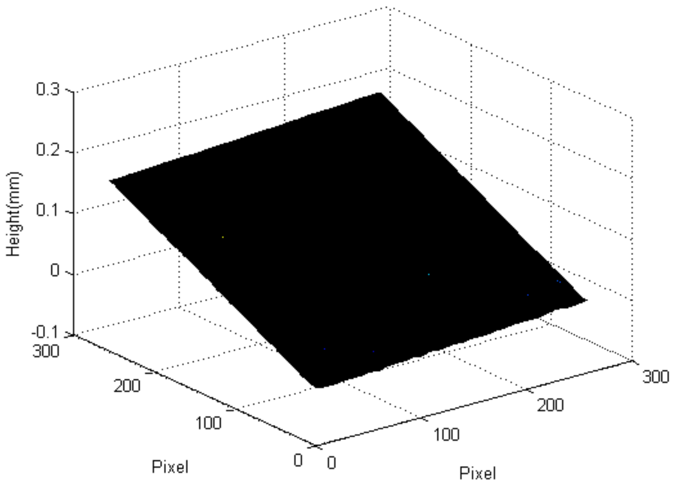
1.2
1
0.8
0.6
0.4
0.2
0
Error(mm)
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
Height(mm)

(d)

Fig.3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0.2
0.1
0
-0.1
Height(mm)
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100
0
Pixel
0
100
200
300
Pixel