# A low cost \& efficient volumetric-error measurement method for five-Axis machine tools 

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#### Abstract

Error compensation is an effective and inexpensive way that can further enhance the machining accuracy of a multiaxis machine tool. The volumetric error measurement method is an essential of the error compensation method. The measurement of volumetric errors of a 5-axis machine tool is very difficult to be conducted due to its complexity. In this study, a volumetric-error measurement method using telescoping ball-bar was developed for three types of 5axis machines. With the use of three derived error models and the two-step measurement procedures, the method can quickly determine the volumetric errors of 5-axis machine tools. Comparing to the measurement methods current used, the proposed method provides the advantages of low cost, high efficiency, easy setup, and high accuracy.


## NOMENCLATURE

$\mathrm{x}_{\mathrm{P} 2}, \mathrm{y}_{\mathrm{P} 2}, \mathrm{z}_{\mathrm{P} 2}, \mathrm{x}_{\mathrm{P} 3}, \mathrm{y}_{\mathrm{P} 3}, \mathrm{z}_{\mathrm{P} 3}=$ coordinates of neighboring points
$\mathrm{x}_{0}, \mathrm{y}_{0}, \mathrm{z}_{0}=$ coordinates of a reference point
$\Delta x, \Delta y, \Delta z=$ total position errors
$1_{n, i}=$ the nominal distance error
$\Delta \mathrm{l}_{\mathrm{i}}=$ distance error
$\mathrm{T}_{\alpha}, \mathrm{T}_{\beta}, \mathrm{T}_{\gamma}=$ homogeneous transformation matrix for rotation
$\mathrm{x}_{2}, \mathrm{y}_{2}, \mathrm{z}_{2}=$ coordinates of the tool tip
$\Delta \mathrm{x}_{\mathrm{V} 1}, \Delta \mathrm{y}_{\mathrm{V} 1}, \Delta \mathrm{z}_{\mathrm{V} 1}=$ total position errors of the tool tip
$\Delta \mathrm{x}_{\mathrm{V} 2}, \Delta \mathrm{y}_{\mathrm{V} 2}, \Delta \mathrm{z}_{\mathrm{V} 2}=$ total position of the machine at $\left(\mathrm{x}_{2}, \mathrm{y}_{2}, \mathrm{z}_{2}\right)$.
$\Delta \alpha, \Delta \beta=$ orientation errors
$\alpha, \beta, \gamma=$ rotational angles
$\mathrm{x}_{2}, \mathrm{y}_{2}, \mathrm{z}_{2}, \mathrm{x}_{\mathrm{t} 2}, \mathrm{y}_{\mathrm{t} 2}, \mathrm{z}_{\mathrm{t} 2}=$ nominal coordinates of reference points
$\Delta \mathrm{x}_{\mathrm{tV} 1}, \Delta \mathrm{y}_{\mathrm{tV} 1}, \Delta \mathrm{z}_{\mathrm{tV} 1}=$ total position errors of reference points
$\Delta \mathrm{x}_{\mathrm{t} 2}, \Delta \mathrm{y}_{\mathrm{t} 2}, \Delta \mathrm{z}_{\mathrm{t} 2}=$ translation errors of the machine table.
$\mathrm{T}_{\mathrm{Z}},=$ homogeneous transformation matrix between two centers

## 1. Introduction

Because of the trend of modern industry, in-use precision five-axes machine tools have increased over triple, and the application for industrial circles have significantly increased.

Because of the tight tolerance of products, in addition to the degrees of freedom for machining, the machining accuracy is the key factor to show the capability and possible application of a five-axis machine tool. Volumetric error compensation technique has been recognized as an effective way to further improve the accuracy of multi-axis machine tools. In error compensation process, knowing the values of errors prior to compensation is necessary. Thus, a method that can effectively determine volumetric errors of a 5 -axis machine tool is an essential of the error compensation technique. The existed measurement method requires an expensive instrument and has complex measurement process. In this study, a volumetric error measurement method with characteristics of low cost, easy setup, and high efficiency was developed for 5 -axis CNC machine tools.

Many researches had been carried out for machine accuracy tests. Bryan [1, 2] developed the magnetic ball-bar to obtain the total position error of a machine at various points. Although the test is not complete for all types of errors, it is quick and easy to perform, and gives good estimates for some of the error components. Two versions of the Magnetic Ball Bar [MBB], and a simple method for testing coordinate measuring machines and machine tools, have been developed at the Lawrence Livermore National Laboratory. The methods were intended to replace the circular comparison standard of the circular test for machine tools. Compared with the standard discs used in the circular tests, the TMBB is more cost efficient, easier to use, and more accurate. Based on the assumption that machines were rigid bodies, Ehmann [3] used Homogeneous Transformation Matrix to develop geometry error models for
multi-axis machines. Kiridena and Ferreira [4] used HTM to build individual mathematics model of geometry errors for common 5 -axis mechanism. Sakamoto [5] used double ball-bar (DBB) to measure geometry errors for 5 -axis machine, and found the values of individual error by the inference of mathematics error model of measurement. Most of the published measurement methods were aimed at measuring error components of an individual axis of motion that are subsequently used in association with the machine error models to evaluate the volumetric errors. However, these measurements cannot be directly used for error compensation. Based on the assumption that points in machine's workspace located close to each other within a small volume exhibit the same total position errors, Wang[6, 7] developed the single socket method (SSM). By using ball-bar, the method provides the capability of directly measuring the total position errors of the machine at discrete points in its workspace. The measurements can be directly used for cutting trajectory error compensation. However, the method is still unable to identify the orientation errors of a machine. To perform the error compensation for a 5-axis machine, both of the total position errors and orientation errors of the machine need to be determined. Most current measurement techniques for 5axis machines are only for single axis calibration, and usually require expensive measurement instruments, such as laser measurement equipment etc. In addition, it is usually very timeconsuming for instrument setup for continuously measuring the machine errors when the machine moves with 5-DOF motion. In this study, by extending the previous research outcome-SSM, a new volumetric errors measurement method incorporating with error models and a 2 -step measurement procedures, was developed for three types of 5 -axis machine tools. Errors measured by this method can be directly used for error compensation [8]. The instrument used in this method is a telescoping ball bar system which is much cheaper than other precision measurement instrument. Thus, the proposed method offers the advantages of low cost, easy setup, and high efficiency for implementation in industry.

In this paper, the measurement principle and the error models for three types of 5-axis machines for error measurement are presented in section 2. The algorithm and measurement procedure of the method are addressed in section 3. In section 4, both the simulation analysis and experiments results were discussed. Finally, conclusions are given in section 5.

## 2. Measurement Principle

5-axis machines compose of three translation axes (T) and two rotating axes $(\mathrm{R})$. According to the configuration of machine, three major types of 5 -axis machine tools (Fig. 1) are widely used for industry: (1) RRTTT type - two rotation axes attached to the machine spindle and three translation axes for the movements of table and spindle housing; (2) TTTRR type three translation axes ( $x, y$, and $z$ axis) for the movements of table and spindle housing, and two rotation axes attached to the table; (3) RTTTR type - one rotation axis attached to machine spindle, three translation axes ( $\mathrm{x}, \mathrm{y}$, and z axis) for the movements of table and spindle housing, and one rotation axis attached to the table. The volumetric errors of 5-axis machine tools include three total position errors and two rotation errors.

To measure the volumetric errors of a 5 -axis machine tool,
the 5-DOF movement of the machine is divided into two consecutive movements: a 3-DOF translation and a 2-DOF rotation. First, the machine moves for the translation movement, and SSM is employed to measure position errors at this location. Then, the machine moves for the rotation movement, and SSM is employed again to measure the position errors at this location. The errors measured at this location are the total position errors caused by the 5-DOF movements. The orientation errors of the machine can then be determined by substituting the two sets of total position errors into a derived orientation error models.


Fig. 1. Schematic of three types of five-axis machine tools

### 2.1 Error Models

### 2.1.1 Total Position Error Models

Figure 2 shows the fundamental concept of $\operatorname{SSM}[6,7]$. When a reference point ( $\mathrm{x}_{0}, \mathrm{y}_{0}, \mathrm{z}_{0}$ ) is selected, total position errors $(\Delta x, \Delta y, \Delta z)$ at point $\left(\mathrm{x}_{\mathrm{P}}, \mathrm{y}_{\mathrm{Pl}}, \mathrm{z}_{\mathrm{Pl}}\right)$ can be determined by
$\left[\begin{array}{c}\Delta x \\ \Delta y \\ \Delta z\end{array}\right]=\left[\begin{array}{ccc}x_{P 1}-x_{0} & y_{P 1}-y_{0} & z_{P 1}-z_{0} \\ x_{P 2}-x_{0} & y_{P 2}-y_{0} & z_{P 2}-z_{0} \\ x_{P 3}-x_{0} & y_{P 3}-y_{0} & z_{P 3}-z_{0}\end{array}\right]^{-1}\left[\begin{array}{l}l_{n, 1} \cdot \Delta l_{1} \\ l_{n, 2} \cdot \Delta l_{2} \\ l_{n, 3} \cdot \Delta l_{3}\end{array}\right]$
Where $\left(\mathrm{x}_{\mathrm{P} 2}, \mathrm{y}_{\mathrm{P} 2}, \mathrm{z}_{\mathrm{P} 2}\right)$ and $\left(\mathrm{x}_{\mathrm{P} 3}, \mathrm{y}_{\mathrm{P} 3}, \mathrm{z}_{\mathrm{P} 3}\right)$ are the coordinates of the two neighboring points. $l_{n, i}$ and $\Delta l_{i}(i=1,2,3)$ respectively represent the nominal distance and distance error between ( $\mathrm{x}_{0}, \mathrm{y}_{0}, \mathrm{z}_{0}$ ) and the three points, ( $\mathrm{x}_{\mathrm{P}}, \mathrm{y}_{\mathrm{P}}, \mathrm{z}_{\mathrm{P} 1}$ ), ( $\mathrm{x}_{\mathrm{P} 2}$, $\left.\mathrm{y}_{\mathrm{P} 2}, \mathrm{z}_{\mathrm{P} 2}\right)$, and ( $\mathrm{x}_{\mathrm{P} 3}, \mathrm{y}_{\mathrm{P} 3}, \mathrm{z}_{\mathrm{P} 3}$ ). The distance error $\Delta \mathrm{l}_{\mathrm{i}}(\mathrm{i}=1,2,3)$ can be directly measured by using ball-bar. Equation 1 is the total position error model. It can be used to determine the total position errors of a multi-axis machine tool with the advantages of low cost, easy setup, and quick measurement.


Fig . 2 The fundamental concept of single socket method

### 2.1.2 Orientation Error Models

With use of differential homogeneous transformation matrix method, the orientation error models that can fast determine the rotation errors of the machine are derived for the three types of 5-axis machine tools.

### 2.1.2.1 Error Model for a RRTTT-type machine tool

With respect to the global machine coordinate frame ( $\mathrm{x}, \mathrm{y}$, $\mathrm{z})_{\mathrm{m}}$, the RRTTT machine can move along the x -, y - and z -axis, and the machine spindle can simultaneously rotate about the x axis and $y$-axis. The coordinate frames for each joint are assigned as shown in the figure. When the machine moves along $\mathrm{x}-, \mathrm{y}$ - and z -axis for $\mathrm{x}_{\mathrm{m}}, \mathrm{y}_{\mathrm{m}}$, and $\mathrm{z}_{\mathrm{m}}$, and the machine spindle rotates about x -axis for an angle of $\alpha$ and about y -axis for an angle of $\beta$, the nominal coordinate of the tool tip, $[\mathrm{P}]_{1}^{\mathrm{M}, \text { Nominal }}$, with respect to the global machine coordinate frame, $(x, y, z)_{m}$, can be expressed with homogeneous transformation matrix as
$[P]_{1}^{M, \text { Nominal }}=T_{x y z} T_{\alpha} T_{\beta}[P]^{S}$
Where $[P]^{\mathrm{S}}=\left[\begin{array}{lll}0 & 0 & \mathrm{Z}_{\mathrm{r}}\end{array} 1^{\mathrm{T}}\right.$ represents the coordinates of tool tip with respect to the coordinate frame of the spindle, $(\mathrm{x}, \mathrm{y}, \mathrm{z})_{\mathrm{s}} . \mathrm{T}_{\mathrm{xyz}}$ represents the homogeneous transformation matrix of $(x, y, z)_{s}$ w.r.t. ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) m and can be expressed as
$T_{x y z}=\operatorname{Tran}\left(x_{m}, x_{s}\right) \operatorname{Tran}\left(y_{m}, y_{s}\right) \operatorname{Tran}\left(z_{m}, z_{s}\right)$
$\mathrm{T}_{\alpha}$ and $\mathrm{T}_{\beta}$ represent the homogeneous transformation matrix of the two rotation movements of the spindle w.r.t. $(\mathrm{x}, \mathrm{y}, \mathrm{z})_{\mathrm{m}}$. When errors exist, the homogeneous transformation matrix becomes

$$
\begin{align*}
& {[P]_{1}^{M, \text { Actual }}=\left[\begin{array}{cccc}
1 & 0 & 0 & x_{1}+\Delta x_{V 1} \\
0 & 1 & 0 & y_{1}+\Delta y_{V 1} \\
0 & 0 & 1 & z_{1}+\Delta z_{V 1} \\
0 & 0 & 0 & 1
\end{array}\right]} \\
& =\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & \cos (\alpha+\Delta \alpha) & -\sin (\alpha+\Delta \alpha) & 0 \\
0 & \sin (\alpha+\Delta \alpha) & \cos (\alpha+\Delta \alpha) & 0 \\
0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{ccc}
\cos (\beta+\Delta \beta) & 0 & \sin (\beta+\Delta \beta) \\
0 & 1 & 0 \\
-\sin (\beta+\Delta \beta) & 0 & \cos (\beta+\Delta \beta) \\
0 & 0 & 0
\end{array}\right]\left[\begin{array}{l}
0 \\
0 \\
0 \\
r_{r} \\
1
\end{array}\right] \\
& =\left[\begin{array}{c}
x_{1}+\Delta x_{V 1}+z_{r} \sin (\beta+\Delta \beta) \\
y_{1}+\Delta y_{V 1}-z_{r} \cos (\beta+\Delta \beta) \sin (\alpha+\Delta \alpha) \\
z_{1}+\Delta z_{V 1}+z_{r} \cos (\alpha+\Delta \alpha) \cos (\beta+\Delta \beta) \\
1
\end{array}\right] \tag{6}
\end{align*}
$$

Equation (6) can also be expressed as
$[P]_{1}^{M, \text { Actual }}=\left[\begin{array}{c}x_{2} \\ y_{2} \\ z_{2} \\ 1\end{array}\right]+\left[\begin{array}{c}\Delta x_{V 2} \\ \Delta y_{V 2} \\ \Delta z_{V 2} \\ 1\end{array}\right]=\left[\begin{array}{c}x_{2}+\Delta x_{V 2} \\ y_{2}+\Delta y_{V 2} \\ z_{2}+\Delta z_{V 2} \\ 1\end{array}\right]$

In Eq. (6) and (7), $x_{2}, y_{2}$, and $z_{2}$ are the coordinates of the nominal position of the tool tip w.r.t. $(\mathrm{x}, \mathrm{y}, \mathrm{z})_{\mathrm{m}}$ after 5 -DOF movements is made. $\Delta \mathrm{x}_{\mathrm{V} 1}, \Delta \mathrm{y}_{\mathrm{V} 1}$, and $\Delta \mathrm{z}_{\mathrm{V} 1}$ represent the total position errors of the tool tip caused by the three translational movements w.r.t. $(\mathrm{x}, \mathrm{y}, \mathrm{z})_{\mathrm{s}}$. $\left(\Delta \mathrm{x}_{\mathrm{V} 2}, \Delta \mathrm{y}_{\mathrm{V} 2}, \Delta \mathrm{z}_{\mathrm{V} 2}\right)$ and $(\Delta \alpha, \Delta \beta)$ are respectively the total position and orientation errors of the machine at the final position $\left(\mathrm{x}_{2}, \mathrm{y}_{2}, \mathrm{z}_{2}\right)$.

By converting the cosine and sine terms in Eq. (6) and (7) to Taylor extension series, and neglecting the higher order terms, the orientation errors can be obtained as

$$
\begin{align*}
& \Delta \alpha=\frac{\left(-y_{1}+y_{2}-\Delta y_{V 1}+\Delta y_{V 2}\right) \cos (\alpha)+\left(-z_{1}+z_{2}-\Delta z_{V 1}+\Delta z_{V 2}\right) \sin (\alpha)}{\left(z_{1}-z_{2}+\Delta z_{V 1}-\Delta z_{V 2}\right) \cos (\alpha)+\left(-y_{1}+y_{2}-\Delta y_{V 1}+\Delta y_{V 2}\right) \sin (\alpha)}  \tag{8}\\
& \Delta \beta=-\frac{\sec (\beta)\left(x_{1}-x_{2}+\Delta x_{V 1}-\Delta x_{V 2}+z_{r} \sin (\beta)\right)}{z_{r}} \tag{9}
\end{align*}
$$

Using Eq. (8) and (9), when $\Delta \mathrm{x}_{\mathrm{V} 1}, \Delta \mathrm{y}_{\mathrm{V} 1}, \Delta \mathrm{z}_{\mathrm{V} 1}, \Delta \mathrm{x}_{\mathrm{V} 2}, \Delta \mathrm{y}_{\mathrm{V} 2}$, and $\Delta \mathrm{z}_{\mathrm{V} 2}$ are measured by SSM (Eq. (1)), the orientation errors of the cutter can be determined.

### 2.1.2.2 Error Model for TTTRR-type Machine tools

For a TTTRR machine tool, the machine table can simultaneously rotate about x -axis and z -axis. The two rotation centers locate at different locations and there is a fixed $z$ direction distance between the two centers. With the similar derivative process described in previous section, the nominal coordinates of a reference point set on the machine table with respect to the coordinate frame, $(\mathrm{x}, \mathrm{y}, \mathrm{z})_{\mathrm{m}}$, can be expressed as
$\begin{aligned} & {[P]_{2}^{M, \text { Nominal }}=T_{x y z} T_{\alpha} T_{z} T_{\gamma}[P]^{\beta_{z}} } \\ = & {\left[\begin{array}{cccc}1 & 0 & 0 & x_{1} \\ 0 & 1 & 0 & y_{1} \\ 0 & 0 & 1 & z_{1} \\ 0 & 0 & 0 & 1\end{array}\right]\left[\begin{array}{cccccc}1 & 0 & 0 & 0 \\ 0 & \cos (\alpha) & -\sin (\alpha) & 0 \\ 0 & \sin (\alpha) & \cos (\alpha) & 0 \\ 0 & 0 & 1\end{array}\right]\left[\begin{array}{llll}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z_{\alpha \gamma} \\ 0 & 0 & 0 & 1\end{array}\right]\left[\begin{array}{cccc}\cos (\gamma) & -\sin (\gamma) & 0 & 0 \\ \sin (\gamma) & \cos (\gamma) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]\left[\begin{array}{l}x_{r} \\ y_{r} \\ z_{r} \\ 1\end{array}\right] }\end{aligned}$ (10)

Where $[P]^{R z}=\left[x_{r} y_{r} z_{r} 1\right]^{T}$ are the coordinates of the reference point with respect to the coordinate frame set at the rotation center on z -axis. $\mathrm{T}_{\mathrm{xyz}}=\operatorname{Tran}\left(\mathrm{x}_{\mathrm{m}}, \mathrm{x}_{\mathrm{b}}\right) \operatorname{Tran}\left(\mathrm{y}_{\mathrm{m}}, \mathrm{y}_{\mathrm{b}}\right) \operatorname{Tran}\left(\mathrm{z}_{\mathrm{m}}, \mathrm{z}_{\mathrm{b}}\right)$, is the homogeneous transformation matrix of $(x, y, z)_{b}$ w.r.t. ( $x, y$, $\mathrm{z})_{\mathrm{m}}$, and the coordinate frame of machine table is set at the rotation center on x -axis. $\mathrm{T}_{\mathrm{z}}$, represents the homogeneous transformation matrix between the frames at the rotation center on z -axis and at the rotation center x-axis. $\mathrm{T}_{\alpha}$ and $\mathrm{T}_{\gamma}$ are the homogeneous transformation matrices for the two rotational movements. $\alpha$ and $\gamma$ are the rotational angles about x -axis and z axis, respectively.

Based on Eq. (10), when errors existed, the actual coordinates of reference point w.r.t. $(x, y, z)_{m}$ become

$$
\begin{align*}
& {[P]_{2}^{M, A \text { ctual }}=\left[\begin{array}{llll}
1 & 0 & 0 & x_{1}+\Delta x_{V 1} \\
0 & 1 & 0 & y_{1}+\Delta y_{V 1} \\
0 & 0 & 1 & z_{1}+\Delta z_{V 1} \\
0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & \cos (\alpha+\Delta \alpha) & -\sin (\alpha+\Delta \alpha) & 0 \\
0 & \sin (\alpha+\Delta \alpha) & \cos (\alpha+\Delta \alpha) & 0 \\
0 & 0 & 0 & 1
\end{array}\right]} \\
& {\left[\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & z_{\alpha \gamma} \\
0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{cccc}
\cos (\gamma+\Delta \gamma) & -\sin (\gamma+\Delta \gamma) & 0 & 0 \\
\sin (\gamma+\Delta \gamma) & \cos (\gamma+\Delta \gamma) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{c}
x_{r} \\
y_{r} \\
z_{r} \\
1
\end{array}\right]} \\
& =\left[\begin{array}{c}
x_{1}+\Delta x_{V 1}+x_{r} \cos (\gamma+\Delta \gamma)-y_{r} \sin (\gamma+\Delta \gamma) \\
y_{1}+\Delta y_{V 1}+x_{r} \cos (\alpha+\Delta \alpha) \sin (\gamma+\Delta \gamma)+y_{r} \cos (\alpha+\Delta \alpha) \cos (\gamma+\Delta \gamma)-\left(z_{r}+z_{\alpha \gamma}\right) \sin (\alpha+\Delta \alpha) \\
z_{1}+\Delta z_{V 1}+x_{r} \sin (\alpha+\Delta \alpha) \sin (\gamma+\Delta \gamma)+y_{r} \cos (\gamma+\Delta \gamma) \sin (\alpha+\Delta \alpha)+\left(z_{r}+z_{\alpha \gamma}\right) \cos (\alpha+\Delta \alpha) \\
1
\end{array}\right. \tag{11}
\end{align*}
$$

Assume that

$$
[P]_{2}^{\mathrm{M}, \text { Actual }}=\left[\begin{array}{c}
x_{2}+\Delta x_{V 2}  \tag{12}\\
y_{2}+\Delta y_{V 2} \\
z_{2}+\Delta z_{V 2} \\
1
\end{array}\right]
$$

In Eq. (11) and (12), $\left(\mathrm{x}_{2}, \mathrm{y}_{2}, \mathrm{z}_{2}\right)$ are the nominal coordinates of the reference point w.r.t. $(x, y, z)_{m}$ after the 5 -DOF movements are made. $\left(\Delta \mathrm{x}_{\mathrm{V} 2}, \Delta \mathrm{y}_{\mathrm{V} 2}, \Delta \mathrm{z}_{\mathrm{V} 2}\right)$ and $(\Delta \alpha, \Delta \gamma)$ are the total position and orientation errors of the machine w.r.t. ( $x, y$, $\mathrm{z})_{\mathrm{m}}$ caused by the 5-DOF movements. $\left(\Delta \mathrm{x}_{\mathrm{V} 1}, \Delta \mathrm{y}_{\mathrm{V} 1}, \Delta \mathrm{z}_{\mathrm{V} 1}\right)$ are the total position errors of the machine w.r.t. $(\mathrm{x}, \mathrm{y}, \mathrm{z})_{\mathrm{b}}$ caused by three translational movements, and they can be directly measured by SSM with setting the reference point on the machine table. Converting the cosine and sine terms in Eq. (11) and (12) to Taylor extension series and neglecting the higher order terms, the orientation errors can be expressed as

$$
\begin{align*}
& \Delta \gamma=\frac{x_{1}-x_{2}+\Delta x_{V 1}-\Delta x_{V 2}+x_{r} \cos (\gamma)-y_{r} \sin (\gamma)}{y_{r} \cos (\gamma)+x_{r} \sin (\gamma)}  \tag{13}\\
& \Delta \alpha=\frac{\left(y_{1}-y_{2}+\Delta y_{V 1}-\Delta y_{V 2}-\left(z_{r}+z_{\alpha \gamma}\right) \sin (\alpha)\right)\left(y_{r} \cos (\gamma)+x_{r} \sin (\gamma)\right)+}{\left(z_{r}+z_{\alpha \gamma}\right) \cos (\alpha)\left(y_{r} \cos (\gamma)+x_{r} \sin (\gamma)\right)+} \\
& \frac{\cos (\alpha)\left(x_{r}^{2}+y_{r}^{2}+\left(x_{1}-x_{2}+\Delta x_{V 1}-\Delta x_{V 2}\right)\left(x_{r} \cos (\gamma)-y_{r} \sin (\gamma)\right)\right)}{\sin (\alpha)\left(x_{r}^{2}+y_{r}^{2}+\left(x_{1}-x_{2}+\Delta x_{V 1}-\Delta x_{V 2}\right)\left(x_{r} \cos (\gamma)-y_{r} \sin (\gamma)\right)\right)} \tag{14}
\end{align*}
$$

Using Eq. (13) and (14), when the total position errors $\Delta \mathrm{x}_{\mathrm{V} 1}, \Delta \mathrm{y}_{\mathrm{V} 1}, \Delta \mathrm{z}_{\mathrm{V} 1}, \Delta \mathrm{x}_{\mathrm{V} 2}, \Delta \mathrm{y}_{\mathrm{V} 2}$, and $\Delta \mathrm{z}_{\mathrm{V} 2}$ are determined by SSM, the orientation errors can be calculated.

### 2.1.2.3 Error Model for a RTTTR-type machine tool

For a RTTTR 5-axis machine tool, the machine spindle and the machine table can rotate about $y$-axis and $x$-axis, respectively. With use of the homogeneous transformation matrix, when the machine spindle makes 4 -DOF movements, the nominal coordinate of the tool tip w.r.t. $(x, y, z)_{m}$ can be expressedas

$$
\begin{align*}
& {[P]_{3}^{M, \text { Nominal }}=T_{x y z} T_{\beta}[P]^{S}} \\
& =\operatorname{Tran}\left(x_{m}, x_{s}\right) \operatorname{Tran}\left(y_{m}, y_{s}\right) \operatorname{Tran}\left(z_{m}, z_{s}\right) \operatorname{Rot}\left(y_{m}, \beta\right)[P]^{S} \tag{15}
\end{align*}
$$

Where $[P]^{S}=\left[\begin{array}{llll}0 & 0 & z_{r} & 1\end{array}\right]^{T}$ represents the coordinates of tool tip w.r.t. ( $\mathrm{x}, \mathrm{y}, \mathrm{z})_{\mathrm{s}} . \mathrm{T}_{\mathrm{xyz}}$ represents the homogeneous transformation matrix between the coordinate frame ( $\mathrm{x}, \mathrm{y}, \mathrm{z})_{\mathrm{s}}$ and $(\mathrm{x}, \mathrm{y}, \mathrm{z})_{\mathrm{m}}$, and $T_{\beta}$ is the homogeneous transformation matrix representing the rotation about y -axis w.r.t. $(\mathrm{x}, \mathrm{y}, \mathrm{z})_{\mathrm{m}}$. $\beta$ is the rotation angle at machine spindle.

After making 4-DOF movements, the nominal coordinate of the reference point set on the machine table w.r.t. $(\mathrm{x}, \mathrm{y}, \mathrm{z})_{\mathrm{m}}$ can be expressed as
$[P]_{t 3}^{M, \text { Nominal }}=T_{x y z} T_{\alpha}[P]^{B}$
$=\operatorname{Tran}\left(x_{m}, x_{b}\right) \operatorname{Tran}\left(y_{m}, y_{b}\right) \operatorname{Tran}\left(z_{m}, z_{b}\right) \operatorname{Rot}\left(x_{m}, \alpha\right)[P]^{B}$
Where $[P]^{B}=\left[x_{t r} y_{t r} \mathrm{Z}_{\mathrm{tr}} 1\right]^{\mathrm{T}}$ represents the nominal coordinate of the reference point w.r.t. $(x, y, z)_{b}$ after three translation movements are made. $\mathrm{T}_{\mathrm{xyz}}$ is the homogeneous transformation matrix between the coordinate frames $(\mathrm{x}, \mathrm{y}, \mathrm{z})_{\mathrm{b}}$ and $(\mathrm{x}, \mathrm{y}, \mathrm{z})_{\mathrm{m}} \cdot \mathrm{T}_{\alpha}$ is the homogeneous transformation matrix for the rotation movement of the machine table.

As the similar derivation in the previous sections, when errors exist Eq. (15) becomes
$[P]_{3}^{M, \text { Actual }}=\left[\begin{array}{c}x_{1}+\Delta x_{V 1}+z_{r} \sin (\beta+\Delta \beta) \\ y_{1}+\Delta y_{V 1} \\ z_{1}+\Delta z_{V 1}+z_{r} \cos (\beta+\Delta \beta) \\ 1\end{array}\right]=\left[\begin{array}{c}x_{2}+\Delta x_{V 2} \\ y_{2}+\Delta y_{V 2} \\ z_{2}+\Delta z_{V 2} \\ 1\end{array}\right]$
Where $[\mathrm{P}]_{3}{ }^{\mathrm{M}, \text { Actual }}$ is the actual coordinate of the tool tip. $\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right.$, $\left.\mathrm{z}_{2}\right)$ and $\left(\Delta \mathrm{x}_{\mathrm{V} 2}, \Delta \mathrm{y}_{\mathrm{v} 2}, \Delta \mathrm{z}_{\mathrm{v} 2}\right)$ are respectively the nominal coordinates and the total position errors of the tool tip w.r.t. (x, $\mathrm{y}, \mathrm{z})_{\mathrm{m}}$ after 4-DOF movements are made. $\left(\Delta \mathrm{x}_{\mathrm{v} 1}, \Delta \mathrm{y}_{\mathrm{v} 1}, \Delta \mathrm{z}_{\mathrm{v} 1}\right)$ are the total position errors of the tool tip w.r.t. $(\mathrm{x}, \mathrm{y}, \mathrm{z})_{\mathrm{s}}$ after the three translational movements are made. $\Delta \beta$ is the orientation error of the machine spindle. When errors exist, Eq. (16) becomes
$[P]_{t 3}^{M, A \text { ctual }}=\left[\begin{array}{c}x_{t 1}+\Delta x_{t V 1}+x_{t r} \\ y_{t 1}+\Delta y_{t V 1}+y_{t r} \cos (\alpha+\Delta \alpha)-z_{t r} \sin (\alpha+\Delta \alpha) \\ z_{t 1}+\Delta z_{t V 1}+y_{t r} \sin (\alpha+\Delta \alpha)+z_{t r} \cos (\alpha+\Delta \alpha) \\ 1\end{array}\right]=\left[\begin{array}{c}x_{t 2}+\Delta x_{t V 2} \\ y_{t 2}+\Delta y_{t V 2} \\ z_{t 2}+\Delta z_{t V 2} \\ 1\end{array}\right]$
Where $[\mathrm{P}]_{\mathrm{t}}{ }^{\mathrm{M}, \text { Actual }}$ is the actual coordinate of reference point. ( $\mathrm{xt} 2, \mathrm{yt} 2, \mathrm{zt} 2$ ) and ( $\Delta \mathrm{xt} 2, \Delta \mathrm{yt} 2, \Delta \mathrm{zt} 2$ ) are respectively the nominal coordinate and the total position errors of the reference point w.r.t. $(\mathrm{x}, \mathrm{y}, \mathrm{z})_{\mathrm{m}}$ after the 4 -DOF movement are made. $\left(\Delta \mathrm{x}_{\mathrm{tV} 1}, \Delta \mathrm{y}_{\mathrm{tV} 1}, \Delta \mathrm{z}_{\mathrm{t} \mathrm{V}_{1}}\right)$ are the total position errors of the reference point w.r.t. $(\mathrm{x}, \mathrm{y}, \mathrm{z})_{\mathrm{b}}$ after the three translation movements are made. $\left(\Delta \mathrm{x}_{12}, \Delta \mathrm{y}_{\mathrm{t}_{2}}, \Delta \mathrm{z}_{12}\right)$ and $\Delta \alpha$ are respectively the translation and orientation errors of the machine table.

Converting the cosine and sine terms in Eq. (20) and (21) to Taylor extension series and neglecting the higher order terms, the orientation error of the machine spindle $(\Delta \beta)$ and the orientation error of reference point on the machine table $(\Delta \alpha)$ can be expressed as

$$
\begin{equation*}
\Delta \beta=-\frac{\sec (\beta)\left(x_{1}-x_{2}+\Delta x_{V 1}-\Delta x_{V 2}+z_{r} \sin (\beta)\right)}{z_{r}} \tag{19}
\end{equation*}
$$

$$
\begin{equation*}
\Delta \alpha=\frac{y_{t 1}-y_{t 2}+\Delta y_{t V 1}-\Delta y_{t V 2}+y_{t r} \cos (\alpha)-z_{t r} \sin (\alpha)}{z_{t r} \cos (\alpha)+y_{t r} \sin (\alpha)} \tag{20}
\end{equation*}
$$

The the variables, $\left(\mathrm{x}_{1}, \mathrm{y}_{1}, \mathrm{z} 1\right),\left(\mathrm{x}_{2}, \mathrm{y}_{2}, \mathrm{z}_{2}\right),\left(\mathrm{x}_{\mathrm{t} 1}, \mathrm{y}_{\mathrm{t} 1}, \mathrm{z}_{\mathrm{t} 1}\right),\left(\mathrm{x}_{\mathrm{t} 2}\right.$, $\left.\mathrm{y}_{\mathrm{t} 2}, \mathrm{z}_{\mathrm{t} 2}\right), \mathrm{z}_{\mathrm{r}}, \mathrm{z}_{\mathrm{tr}}, \alpha$ and $\beta$, are known value when the machine is driven. With using the single socket method, total position errors $\left(\Delta \mathrm{x}_{\mathrm{V} 1}, \Delta \mathrm{y}_{\mathrm{V} 1}, \Delta \mathrm{z}_{\mathrm{V} 1}\right),\left(\Delta \mathrm{x}_{\mathrm{V} 2}, \Delta \mathrm{y}_{\mathrm{V} 2}, \Delta \mathrm{z}_{\mathrm{V}_{2}}\right)$ of the tool tip can be directly measured. ( $\left.\Delta \mathrm{x}_{\mathrm{tV} 1}, \Delta \mathrm{y}_{\mathrm{tv} 1}, \Delta \mathrm{z}_{\mathrm{tv} 1}\right)$ and ( $\left.\Delta \mathrm{x}_{\mathrm{tV} 2}, \Delta \mathrm{y}_{\mathrm{tV} 2}, \Delta \mathrm{z}_{\mathrm{tV} 2}\right)$ can also be directly measured at the reference point. After substituting all the errors into Eq. (19) and (20), the orientation errors, $\Delta \alpha$ and $\Delta \beta$, of the machine can be determined.

## 3. The Two-step Measurement Procedure

According to the derived error models, it is noted that the orientation errors of a 5 -axis machine tool are functions of two sets of total position errors: (1) the total position errors measured after three translational movements are made; (2) the total position errors measured after the 2-DOF rotation are made. Thus, a two-step measurement procedure incorporating with SSM was developed for determination of the two sets of total position errors. For RRTTT-type machine, the measurement procedures are: (1) Set the magnetic centre mount on machine bed and install ball-bar to the machine. The end of ball-bar attached to machine table is the reference point; (2) Move machine spindle to the first position with three translational movements; (3) Employ SSM to determine the total position errors occurring at the first location, i.e. $\left(\Delta \mathrm{x}_{\mathrm{V} 1}, \Delta \mathrm{y}_{\mathrm{V} 1}, \Delta \mathrm{z}_{\mathrm{V} 1}\right)$ in Eq. (6); (4) Move machine spindle to the second position with two rotational movements; (5) Use SSM to determine the total position errors occurring at the second location, i.e. $\left(\Delta \mathrm{x}_{\mathrm{V} 2}, \Delta \mathrm{y}_{\mathrm{V} 2}\right.$, $\Delta z_{v_{2}}$ ) in Eq. (7); (6) Determine the orientation errors $\Delta \alpha$ and $\Delta \beta$ by substituting the two sets of total position errors into Eq. (8) and (9).

For the TTTRR-type 5 -axis machine, the procedure is similar to the procedure for the RRTTT-type machine, except that the end of ball-bar that mounted on the machine spindle is chosen to be the reference point. Because both two rotation movements are made by turning table and the machine spindle stays stationary, the orientation errors occur at the turning table. By substituting the measured total position errors, $\left(\Delta \mathrm{x}_{\mathrm{V} 1}, \Delta \mathrm{y}_{\mathrm{V} 1}\right.$, $\left.\Delta \mathrm{z}_{\mathrm{V}_{1}}\right)$ and ( $\Delta \mathrm{x}_{\mathrm{V}_{2}}, \Delta \mathrm{y}_{\mathrm{V}_{2}}, \Delta \mathrm{z}_{\mathrm{V}_{2}}$ ), into Eq. (13) and (14), orientation errors, $\Delta \alpha$ and $\Delta \gamma$ can be determined.

For RTTTR-type machine, because the machine spindle and machine table will have 1-DOF rotation separately, the two orientation errors should be determined separately. The measurement procedure for orientation error $\Delta \beta$ is: (1) Set the magnetic centre mount on machine bed, and install ball-bar on the machine as regular use; (2) Move machine spindle to the first location with three translational movements; (3) Employ SSM to measure the total position errors at the first location, i.e. ( $\Delta \mathrm{x}_{\mathrm{V} 1}, \Delta \mathrm{y}_{\mathrm{V} 1}, \Delta \mathrm{z}_{\mathrm{V} 1}$ ) in Eq. (17); (4). Rotate machine spindle to the second location; (5) Set reference point on the turning table, and employ SSM to determine the total errors occurring at the second location, i.e. ( $\Delta \mathrm{x}_{\mathrm{V} 2}, \Delta \mathrm{y}_{\mathrm{V} 2}, \Delta \mathrm{z}_{\mathrm{V} 2}$ ) in Eq. (17); (6) Determine the orientation errors $\Delta \beta$ by substituting ( $\Delta \mathrm{x}_{\mathrm{V} 1}, \Delta \mathrm{y}_{\mathrm{V} 1}$, $\left.\Delta \mathrm{z}_{\mathrm{V}_{1}}\right),\left(\Delta \mathrm{x}_{\mathrm{V}_{2}}, \Delta \mathrm{y}_{\mathrm{V} 2}, \Delta \mathrm{z}_{\mathrm{V}_{2}}\right)$ into Eq. (19).

The measurement procedure for orientation error $\Delta \alpha$ is: (1) Set the magnetic center mount on machine bed and install ballbar on the machine as regular use; (2) Move the turning table to
the first location with three translational movements; (3) Use SSM to determine the total errors occurring at the first location, i.e. ( $\left.\Delta \mathrm{x}_{\mathrm{tV1}}, \Delta \mathrm{y}_{\mathrm{tV} 1}, \Delta \mathrm{z}_{\mathrm{tV1}}\right)$ in Eq. (18); (4). Rotate the turning table to the third location; (5). Set the reference point on the machine spindle, and employ SSM to determine the total errors ( $\Delta \mathrm{x}_{\mathrm{t} \mathrm{V} 2}$, $\Delta \mathrm{y}_{\mathrm{tV} 2}, \Delta \mathrm{z}_{\mathrm{tV} 2}$ ) at the third location in Eq. (18); (6) Determine the orientation errors $\Delta \alpha$ by substituting ( $\left.\Delta \mathrm{x}_{\mathrm{tV} 1}, \Delta \mathrm{y}_{\mathrm{tV} 1}, \Delta \mathrm{z}_{\mathrm{tV} 1}\right),\left(\Delta \mathrm{x}_{\mathrm{t} V 2}\right.$, $\Delta \mathrm{y}_{\mathrm{tV} 2}, \Delta \mathrm{z}_{\mathrm{tV} 2}$ ) into Eq. (20).

To enhance the convenience of use of the proposed measurement method for industry, a computer-aided measurement system integrated with a ball-bar system was developed in Matlab language based on the proposed error models and measurement procedures.

## 4 .Simulation and Experimental Illustrations

In order to verify the proposed measurement method, both computer simulation and experiments with a machine were conducted.

In the simulation, a RRTTT-type machine tool was selected. The total length for spindle and cutter is 250 mm . (i.e. the radius of rotation). Several 5-DOF movements were planned for verification. Each movement is composed of three translations in $\mathrm{x}, \mathrm{y}$, and z direction and two rotations about x and $y$-axis (Rotation angles: $\alpha, \beta$ ). All movements have same translations as x-travel: -50 mm , y-travel: -15 mm , and z -travel: 350 mm . Different rotation combinations and nominal orientation errors were selected as:
(1) $\alpha=30^{\circ}, \beta=0^{\circ}, \Delta \alpha=0.1^{\circ}$ and $\Delta \beta=0^{\circ}$; (2) $\alpha=0^{\circ}, \beta=$ $40^{\circ}, \Delta \alpha=0^{\circ}$ and $\Delta \beta=0.08^{\circ}$; (3) $\alpha=45^{\circ}, \beta=60^{\circ}, \Delta \alpha=0.12^{\circ}$ and $\Delta \beta=0.1^{\circ}$; (4) $\alpha=60^{\circ}, \beta=50^{\circ}, \Delta \alpha=0.15^{\circ}$ and $\Delta \beta=$ $0.09^{\circ}$; (5) $\alpha=30^{\circ}, \beta=60^{\circ}, \Delta \alpha=0.1^{\circ}$ and $\Delta \beta=0.1^{\circ}$. The nominal location after three translations was $(-50,-15,250)$. Based on the calculation of homogeneous transformation matrix and D-H rule, the nominal location of the machine spindle ( $\Delta \mathrm{x} 2$, $\Delta \mathrm{y} 2, \Delta \mathrm{z} 2)$ after rotations are $(-50,110,283.494),(-210.697,-15$, 308.489), (-266.506, 73.388, 411.612), (-241.511, 124.168, 419.652), and (-266.506, 47.5, 391.747). For simplicity, the total position errors caused by translational movements were neglected in this simulation. The total position errors of the machine after rotations were calculated and shown in Table 1. After substituting the total position errors into Eq. (8) and (9), the orientation errors, $\Delta \alpha$ and $\Delta \beta$, were obtained (Table 1). The orientation errors were all very close to the nominal orientation errors. The average difference between the nominal orientation errors and calculated orientation errors were about $0.0001^{\circ}$. It proves that the derived error models are quite accurate.

In the experiments, a TTTRR-type machining center produced by Dah-Lih Machinery Co. was used. Four 5-DOF test movements were conducted. Each test movement includes three translations $(-49.275 \mathrm{~mm}$ along x -axis; -14.312 mm along y axis; - 177.777 mm along $\mathrm{z}-\mathrm{z}$-axis) and a 2 -DOF rotation. Four 2-DOF rotations were selected for experiments: (1) $\alpha=0^{\circ}, \gamma=30^{\circ}$; (2) $\alpha=15^{\circ}, \gamma=0^{\circ}$; (3) $\alpha=10^{\circ}, \gamma=30^{\circ}$; (4) $\alpha=10^{\circ}, \gamma=45^{\circ}$. SSM was employed to respectively measure the total position errors of the machine. After substituting the measured errors into Eq. (13) and (14), the orientation errors were determined. Both of the total position errors and orientation errors were shown in Table 2. The maximum orientation error, $\Delta \alpha=-0.0426^{\circ}$ and $\Delta \gamma=0.0684^{\circ}$
occurred at $\alpha=0^{\circ}, \gamma=30^{\circ}$. It was noted that $\Delta \gamma$ is larger than $\Delta \alpha$. This is because the driving mechanism of the turning table was damaged before. The experimental results reflected the facts.

## 5. Conclusions

A new volumetric error measurement method composing of error models, two-step measurement procedures, and use of SSM was developed for three major types of 5 -axis machine tools in this study. The error models for RRTTT-type, TTTRRtype, and RTTTR-type 5 -axis machine tools were derived. The two-step measurement procedures were developed for standard operation. Simulation and experiments were conducted to verify the proposed measurement method. The results have shown good feasibility and reliability of the proposed method. Without using expensive measurement instrument and complex measurement procedures, the proposed method offers the advantages of low cost, easy set up, and high efficiency for determining the volumetric errors of 5-axis machine tools.

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|  |  | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Rotat. <br> Angles | $\alpha / \beta$ | $30^{\circ} / 0^{\circ}$ | $0^{\circ} / 40^{\circ}$ | $45^{\circ} / 60^{\circ}$ | $60^{\circ} / 50^{\circ}$ | $30^{\circ} / 60^{\circ}$ |
| assumed <br> Angles. | $\Delta \alpha / \Delta \beta$ | $0.1^{\circ} / 0^{\circ}$ | $0^{\circ} / 0.08^{\circ}$ | $0.12^{\circ} / 0.1^{\circ}$ | $0.15^{\circ} / 0.09^{\circ}$ | $0.1^{\circ} / 0.1^{\circ}$ |
| Nomin. Coord.B/f Rotat. $\left(\mathrm{x}_{1}, \mathrm{y}_{1}, \mathrm{z}_{1}\right)(\mathrm{mm})$ |  | $(-50,-15,500)$ |  |  |  |  |
| Total position errors B/f Rotat. $\left(\Delta \mathrm{x}_{V l}, \Delta \mathrm{y}_{V 1}, \Delta \mathrm{z}_{V 1}\right)(\mathrm{mm})$ | $(0,0,0)$ |  |  |  |  |  |
|  | $x_{2}$ | -50 | -210.697 | -266.506 | -241.511 | -266.506 |
|  | $y_{2}$ | 110 | -15 | 73.388 | 124.168 | 47.5 |
| Total <br> position | $\Delta x_{V 2}$ | 0 | -0.267 | -0.218 | -0.252 | -0.218 |
|  | $\Delta y_{V 2}$ | 0.378 | 008.489 | 411.612 | 419.652 | 391.747 |
|  | $\Delta z_{V 2}$ | 0.218 | 0.225 | 0.452 | 0.514 | 0.436 |
| Calculated <br> Error | $\Delta \alpha^{\prime}$ | $0.1^{\circ}$ | $0^{\circ}$ | $0.12^{\circ}$ | $0.15^{\circ}$ | $0.1^{\circ}$ |
|  | $\Delta \beta^{\prime}$ | $0^{\circ}$ | $0.07995^{\circ}$ | $0.09985^{\circ}$ | $0.08992^{\circ}$ | $0.09985^{\circ}$ |

Table 1. The Simulation results of measuring volumetric errors for a RRTTT-type Machine Tool

|  |  | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Rotat. <br> Angle | $\alpha / \gamma$ | $0^{\circ} / 30^{\circ}$ | $15^{\circ} / 0^{\circ}$ | $10^{\circ} / 30^{\circ}$ | $10^{\circ} / 45^{\circ}$ |
|  | Nomin. Coord.B/f rotation $(\mathrm{x}, \mathrm{y}, \mathrm{z})(\mathrm{mm})$ |  | $(-49.275,-14.312,-14.312$ |  |  |  |
| Nomin. <br> Coord.B/f <br> rotation <br> (mm) | $x$ | -35.517 | -49.275 | -35.517 | 24.723 |
|  | $y$ | -37.032 | 32.188 | -5.599 | -13.409 |
|  | $z$ | -177.777 | -175.424 | -181.507 | -182.884 |
| Total <br> position <br> errors B/f <br> Rotation <br> (mm) | $\Delta \mathrm{x}$ | 0.0067 | -0.0104 | -0.1740 | -0.1740 |
|  | $\Delta \mathrm{y}$ | 0.0373 | 0.2396 | -0.1721 | -0.1721 |
| Total <br> position <br> errors After <br> Rotation <br> (mm) | $\Delta \mathrm{x}$ | 0.0519 | 0.0297 | -0.0619 | -0.0619 |
|  | $\Delta \mathrm{y}$ | 0.1255 | -0.0861 | -0.0555 | -0.0180 |
|  | $\Delta \mathrm{z}$ | 0.0259 | -0.0448 | -0.1307 | -0.1116 |
| Calculated <br> Orientation <br> error | $\Delta \alpha$ | $-0.0426^{\circ}$ | $-0.0161^{\circ}$ | $-0.0128^{\circ}$ | $-0.0396^{\circ}$ |
|  | $\Delta \gamma$ | $0.0684^{\circ}$ | $0.4514^{\circ}$ | $-0.2403^{\circ}$ | $-0.2513^{\circ}$ |

Table 2. The experimental results of measuring volumetric errors for a TTTRR-type Machine tool

