Error sources analysis of nonlinearity in laser heterodyne interferometry

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KEYWORDS : Heterodyne Interferometry, Nonlinearity, Error Sources

The first-harmonic and second-harmonic periodic nonlinearities severely restrict the further promotion for the measurement accuracy of heterodyne interferometry. To reduce the nonlinearity of nanometer measurement in heterodyne interferometry, the influence mechanism of error sources upon the nonlinearity must be researched. Based on the generation mechanism of nonlinearity, the models on how first-harmonic and second-harmonic nonlinearity caused by error sources are proposed. The mechanism that all sorts of error sources influence the nonlinearity is analyzed completely. The error sources include laser, the polarization beam splitter, the polarizer, the corner-cube retroreflector, the analyzer and the amplitude variation in coherent transmission. The theoretic foundation is provided accordingly to minimize nonlinearity in heterodyne interferometry.

Manuscript received: January XX, 2011 / Accepted: January XX, 2011

1. Introduction

Heterodyne interferometry nonlinearity existing in the measurement process was presented by Quenelle in 1983 [1]. In recent decades, many scholars have undertaken extensive research[2-8]. The nonlinearity is periodic and it is caused by the beam frequency leakage. The amplitude of the nonlinearity can be a few nanometers, or even more than 10nm. The nonlinearity is the important error source limiting the further improvement for the accuracy of laser heterodyne interferometry.

Heterodyne interferometer nonlinearity includes the firs-harmonic and second-harmonic nonlinearities. From the following aspects of laser heterodyne interferometry, the error sources of the nonlinearity will be analyzed:

(1) Non-orthogonal polarized laser beams.
(2) Elliptical polarized laser beams.
(3) Orientation error and phase retardation error of polarizer.
(4) Rotational alignment error and nonideal splitting performance of polarization beam splitter (PBS).
(5) Nonideal polarization properties of coated cube-corner retro-reflector.
(6) Orientation error of analyzer.
(7) Amplitude variation in coherent transmission.

2. Theory of laser heterodyne interferometry

The laser heterodyne interferometer shown in Fig.1 consists of a laser source with two orthogonal polarized beams and a different frequency (f_1 and f_2). The two beams can be represented as:

$$\vec{E}_1 = \vec{i}E_{01}\sin\left(2\pi f_1 t + \phi_{01}\right) \quad (1)$$

$$\vec{E}_2 = \vec{j}E_{02}\sin\left(2\pi f_2 t + \phi_{02}\right) \quad (2)$$

Where $\vec{i}$ and $\vec{j}$ represent unit vector of $\vec{E}_1$ and $\vec{E}_2$, E_{01} and E_{02} represent amplitude, f_1 and f_2 represent the frequency of $\vec{E}_1$ and $\vec{E}_2$, $\Delta f = f_1 - f_2$, $\phi_{01}$and ϕ_{02} represent phase.

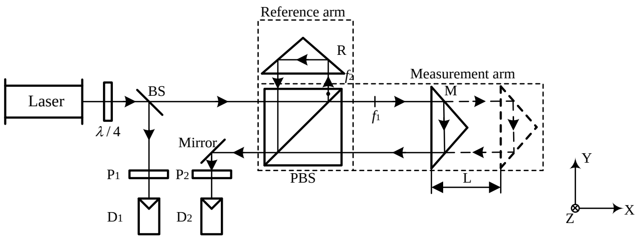

Fig. 1 Laser heterodyne interferometry principle

A non-polarizing beam splitter divides the laser beam into two parts: one reference part and one measurement part. The reference part passes the analyzer and a photo-detector resulting in an interference signal. This signal can be represented as:

$$I_r \sim I_0 \cos\left[2\pi\left(f_1 - f_2\right)t + \phi_0\right] \quad (3)$$

Where $I_0 = E_{01}E_{02}/2$, $\phi_0 = \phi_{01} - \phi_{02}$

The measurement part is divided by a polarizing beam splitter in a reference arm and in a measurement arm. The signal in the measurement arm receives a Doppler shift as a result of the moving corner-cube can be represented as:

$$I_m \sim I_0 \cos\left[2\pi\left(f_1 - f_2\right)t + \phi_0 + \Delta\phi\right] \quad (4)$$

Where $\Delta\phi = 4\pi nL/\lambda$, L represents the displacement of measuring corner-cube M.

Due to a variety of factors, polarization mixing will exist between the two intervention arms. And Doppler frequency shift is caused by the reference beam transmitted into the measuring arm. The measured signal is

$$I_m \sim I_0 \cos\left[2\pi\left(f_1 - f_2\right)t + \phi_0 + \Delta\phi + \Delta\phi_{nonlin}\right] \quad (5)$$

Where $\Delta\phi_{nonlin}$ is the nonlinearity of laser heterodyne interferometer.

3. Error sources analysis of nonlinearity

3.1 Elliptical polarized laser beams

The laser source produces nonideal circular polarization state that depends on the dichroism and the birefringence.

Nonideal circular polarization light is produced by the laser due to the dichrosm and the birefringence of the media in the laser during the transformission. After quarter wave, an elliptically polarized light with certain ellipticity, instead of a linearly polarized light formed. The ellipticity of the laser polarization beams is defined as $d\varepsilon_1$ and $d\varepsilon_2$. It is supposed that $d\varepsilon_1$=0, and then nonlinearity can be represented as:

$$\Delta\phi_{nonlin} = -\arctan\frac{\sin\left(d\varepsilon_2\right)\cos\left(\Delta\phi\right)}{\cos\left(d\varepsilon_2\right) - \sin\left(d\varepsilon_2\right)\sin\left(\Delta\phi\right)} \quad (6)$$

As a result of different ellipticity $d\varepsilon_2$, different nonlinearity simulation curves are shown in Fig.2. It is shown that in Fig.2 the elliptical polarization light introduces an increasing first harmonic nonlinearity (period $\lambda/2$). When $d\varepsilon_2$=5°, the nonlinearity is 4.41nm.

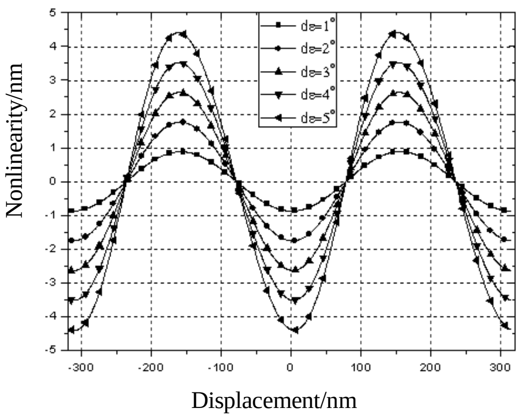

Fig. 2 The nonlinearity resulting from different ellipticity

3.2 Elliptical polarized laser beams

Assuming the beam with frequency f_2 has a deviation α from the orthogonal direction. The nonlinearity caused by non-orthogonal error can be represented as:

$$\Delta\phi_{nonlin} = -\arctan\frac{\sin\alpha\sin\left(\Delta\phi\right)}{\sin\alpha\cos\left(\Delta\phi\right) + \cos\alpha} \quad (7)$$

From the formula (7), the non-orthogonal error introduces a first-harmonic nonlinearity that increases with the increasing of the non-orthogonal error.

3.3 Orientation error and phase retardation error of polarizer

Two opposite left and right circularly polarized rotation light, which are sent out by a laser, have a small frequency difference, and they can be converted to the vibration direction of the two beams of orthogonal linearly polarized by the polarizer.

3.3.1 Orientation error of polarizer

The rotation angle of the polarizer compare to $\pi/4$ is referred to as θ. Circularly polarized light is converted to linearly polarized light by the polarizer, but the vibration direction of linearly polarized light will be different from the direction of PBS. So nonlinearity caused by orientation error of polarizer can be represented as:

$$\Delta\phi_{nonlin} = -\arctan\frac{-\sin^2\theta\sin\left(2\Delta\phi\right)}{\cos^2\theta - \sin^2\theta\cos\left(2\Delta\phi\right)} \quad (8)$$

Simulation curves obtained according to the formula (8) are shown in Figure 3. Orientation error of polarizer introduces a second-harmonic nonlinearity (period $\lambda/4$) which increases with the increasing of the orientation error. When θ=7°, the nonlinearity is 0.76 nm.

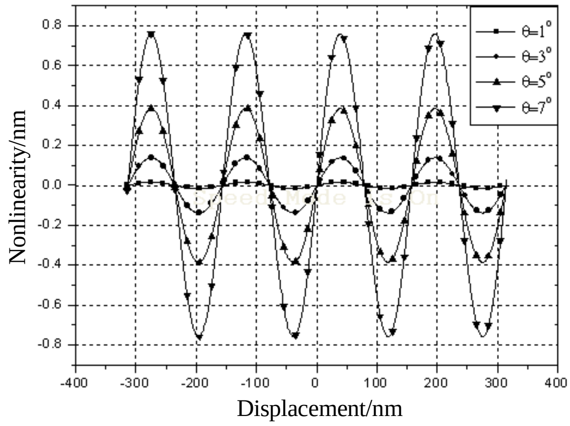

Fig. 3The nonlinearity resulting from different polarizer rotation error

3.3.2 Phase retardation error of polarizer

The δ represents phase retardation error of polarizer. Circular polarized light is converted to elliptically polarized light by the polarizer with phase retardation error. When θ=0°, the nonlinearity caused by phase retardation error of polarizer can be represented as:

$$\Delta\phi_{nonlin} = -\arctan\frac{\frac{\delta^2}{4}\sin\left(2\Delta\phi\right)}{1 + \frac{\delta^2}{4}\cos\left(2\Delta\phi\right)} \quad (9)$$

According to the formula (9), the polarizer with phase retardation error will introduce a second-harmonic nonlinearity that increases with the increasing of the phase retardation error. When δ=7°, the nonlinearity is 0.17 nm.

3.4 Rotational alignment error and nonideal splitting performance of PBS

S polarization light and p polarization light can be completely separated by ideal PBS. Due to rotational alignment error and non-ideal splitting performance of PBS, s polarization light and p polarization light can't be completely separated. So the nonlinearity is caused by nonideal PBS.

3.4.1 Rotational alignment error of PBS

The direction of PBS with rotational alignment error β will be inconsistent with the polarized direction of heterodyne interferometry. The nonlinearity caused by rotational alignment error of PBS can be represented as:

$$\Delta\phi_{nonlin} = -\arctan\frac{-\sin^2\beta\sin\left(2\Delta\phi\right)}{\cos^2\beta - \sin^2\beta\cos\left(2\Delta\phi\right)} \quad (10)$$

According to the formula (10), PBS with rotational alignment error introduces an increasing second-harmonic nonlinearity. When β=8°, the nonlinearity is 1.00 nm.

3.4.2 Nonideal splitting performance of PBS

T_p and R_s respectively represent transmissivity and reflectivity of PBS. Ideally T_p= R_s =1,but actually T_p and R_s are less than 1. Therefore, there will be p polarized light included in the light reflected by PBS, and s polarized light included in the light transmitted by PBS. At the same time $T_s+R_s=1$, $T_p+R_p=1$. The nonlinearrity caused by non ideal splitting performance of PBS can be repre-sented as:

$$\Delta\phi_{nonlin} = -\arctan\frac{[T_p^2(1-R_s)^2+R_s^2(1-T_p)^2]\sin(\Delta\phi)+(1-R_s)^2(1-T_p)^2\sin(2\Delta\phi)}{[T_p^2(1-R_s)^2+R_s^2(1-T_p)^2]\cos(\Delta\phi)+(1-R_s)^2(1-T_p)^2\cos(2\Delta\phi)+T_p^2R_s^2} \tag{11}$$

where $a=T_p^2(1-R_s)^2+R_s^2(1-T_p)^2$, $b=(1-R_s)^2(1-T_p)^2$, $c=T_p^2R_s^2$.

To simplify the analysis, if R_s=1, the simulation curves obtained from the formula (11) are shown in Figure 4. When transmitssivity is nonideal, first-harmonic nonlinearity is introduced. As transmitssivity decreases, nonlinearity would increase. When T_p reduces from 0.99 to 0.90, nonlinearity increases from 0.01 nm to 0.62 nm.

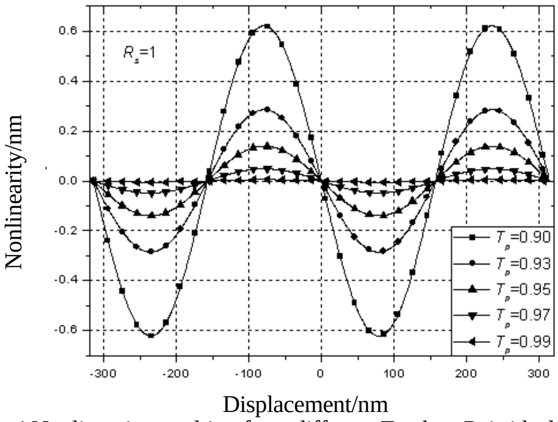

Fig.4 Nonlinearity resulting from different T_p when R_s is ideal

When R_s is nonideal, the simulation curves obtained from the formula (11) are shown in Figure 5. When R_s is 0.90 and T_p reduces form 0.99 to 0.90, nonlinearity increases from 0.62 nm to 1.24 nm. Therefore, transmissivity and reflectivity of PBS have a great influence on the nonlinearity when they become weaker.

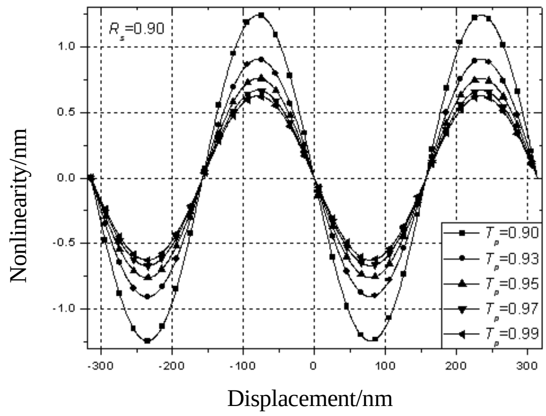

Fig.5 Nonlinearity resulting from different T_p when R_s = 0.9

3.5 Nonideal polarization properties of coated cube-corner retroreflector

A cube-corner is an optical device with three adjacent reflecting surfaces forming the corner of a cube. It is a well-known property that the beam reflected through three internal reflections in a cube-corner is counter parallel to the incident beam irrespective of the considerable tilt of the cube-corner. This advantage makes cube-corners very useful in heterodyne interferometers, and cube-corners are widely used instead of plane mirrors.

The change of polarization property of light can be described in terms of total reflection theory when the linearly polarized collimated beam is incident upon and reflected from a cube-corner[8].

When linearly polarized light incident on the metal-coated solid-state cube-corners, reflected light becomes ellipticcally polarized light. And the ideal linearly polarized lights pass PBS and are respectively reflected by reference cube-corner and measuring cube-corner. Both of lights can be represented as:

$$\vec{E}_2{}' = \vec{i}c_{11}E_{02}\sin(2\pi f_2 t+\phi_{02})+\vec{j}c_{21}E_{02}\sin(2\pi f_2 t+\phi_{02}) \tag{12}$$

$$\vec{E}_1{}' = \vec{i}c_{12}E_{01}\sin(2\pi f_1 t+\phi_{01})+\vec{j}c_{22}E_{01}\sin(2\pi f_1 t+\phi_{01}) \tag{13}$$

Where

$c_{11}=\left(r_s^3+6r_pr_s^2-3r_sr_p^2\right)/8$, $c_{12}=\sqrt{3}r_p\left(r_s+r_p\right)^2/8$,

$c_{21}=-\sqrt{3}r_s\left(r_s+r_p\right)^2/8$, $c_{22}=\left(r_p^3+6r_p^2r_s-3r_pr_s^2\right)/8$,

r_s and r_p respectively represent reflectivity of s polarization light and reflectivity of p polarization light.

The signal in the measurement arm receives a Doppler shift as a result of the moving cube-corner. It can be represented as:

$$\begin{aligned} \mathrm{I_m} &\sim \left\| c_{12}\right|\exp(i\delta_{12})E_{01}\exp i(2\pi f_1 t+\phi_{01}+\Delta\phi)+\left|c_{21}\right|\exp(i\delta_{21})E_{02}\exp i(2\pi f_2 t+\phi_{02})\right|^2 \\ &\sim I_0\left|c_{12}\right|\left|c_{21}\right|\cos\left[2\pi(f_1-f_2)t+\Delta\phi+\delta_{21}-\delta_{12}\right] \end{aligned} \tag{14}$$

where $\left(|c_{12}|,\delta_{12}\right)$, $\left(|c_{21}|,\delta_{21}\right)$ respectively represent amplitude and phase of c_{12} and c_{21}.

Based on the formula (14), the polarization property of cube-corner will not have effect on the nonlinearity of heterodyne interferometry when other error sources don't exist, but have an influence on the signal strength. When other error sources exist, the polarization property of cube-corner will have effect on the nonlinearity.

3.6 Orientation error of analyzer

The two linearly polarized lights described in equation (1) and (2) pass the analyzer with orientation error ϑ. Then the combined beam can be represented as:

$$E = E_x\cos\left(\frac{\pi}{4}-\vartheta\right)+E_y\sin\left(\frac{\pi}{4}-\vartheta\right) \tag{15}$$

The signal in the measurement arm receives a Doppler shift as a result of the moving cube-corner. It can be represented as:

$$I_m \sim E^2 \sim \frac{1}{2}\cos\vartheta I_0 \cos\left[2\pi\left(f_1 - f_2\right)t + \phi_0 + \Delta\phi\right] \tag{16}$$

Based on the formula (16), the orientation error of analyzer will not have effect on the nonlinearity of heterodyne interferometry when other error sources don't exist, but have an influence on the signal strength. When other error sources exist, the orientation error of analyzer will have effect on the nonlinearity

3.7 Amplitude variation in coherent transmission

In the laser heterodyne interferometric measurement, the amplitude is attenuated because of the move of measurement cube-corner retroreflector or the absorption of optic element. The attenuation leads the difference of amplitude between measurement arm and reference arm and affects the nonlinearity when incorrect frequency mixing exists.

The attenuation of laser beam in measurement arm is described by the amplitude factor *k*. The following expressions are the beams that enter measurement arm and reference arm after splitting in the PBS:

$$\vec{E}_1 = \vec{i}kE_{01}\sin\left(2\pi f_1 t + \phi_{01} + \Delta\phi\right) \tag{17}$$

$$\vec{E}_2 = \vec{j}E_{02}\sin\left(2\pi f_2 t + \phi_{02}\right) \tag{18}$$

where $\vec{E}_1$ and $\vec{E}_2$ are recombined in the PBS and detected by photodetector. The measurement signal I_m is obtained:

$$I_m \sim \frac{1}{2}\left(\vec{E}_x + \vec{E}_y\right)^2 \sim kI_0 \cos\left[2\pi\left(f_1 - f_2\right)t + \phi_0 + \Delta\phi\right] \tag{19}$$

Based on the formula (19), amplitude variation in coherent transmission will not have effect on the nonlinearity of heterodyne interferometry when other error sources don't exist, but have an influence on the signal strength. When other error sources exist, amplitude variation in coherent transmission will have effect on the nonlinearity.

4. Conclusions

In summary, there are a variety of error sources of nonlinearity in laser heterodyne interferometry .Nonlinearity usually can be a few nanometers. This is a severe limitation for the further improvement on measurement accuracy of heterodyne interferometry. Based on the above analysis on error sources and their effects on the nonlinearity, solid theoretical basis is founded for a method to reduce and inhibit the nonlinearity.

ACKNOWLEDGEMENT

This work was supported in part by the National Science Foundation of China grant No.50805003.

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