

# Mass estimation of loose part in nuclear power plant based on multiple regression

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*Abstract* According to the application of Hilbert-Huang transform to non-stationary signal and the relation between the mass of loose part and corresponding frequency content, a new method of loose part mass estimation based on marginal Hilbert-Huang spectrum and multiple regression is proposed in this paper. The frequency spectrum of loose part in nuclear power plant can be expressed by marginal Hilbert-Huang spectrum (MHS). The Multiple regression model for mass estimation, which is constructed by the MHS of impact signals, can then be used to predict the unknown masses. Simulated experiment verified that the method is feasible.

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## 1. Introduction

The prime circuit and the pressure vessel of a nuclear power plant is filled with high pressure cooling water for controlling the temperature of the reactor. Suffering from the everlasting scouring of the high pressure as well as high temperature cooling water and the vibration caused by the vibration of the machine activity and the water circulating for long time, the parts inside the reactor are possibly to become loose from their original positions or even drop off. The loosen and dropped parts are brought by the circular cooling water to impact or even damage the other parts of the reactor. For the purpose of securing nuclear power plants, nuclear power plants in European countries are demanded to install Loose Part Monitoring System (LPMS) which is designed to monitor abnormal vibration signals from the reactor. Mass estimation, which is one of the prime functions of LPMS, has been researched to distinguish the mass of the loose part in the purpose of evaluating the fault grade and gaining proof for corresponding strategy.

There are two primary methods for mass estimation traditionally<sup>[1-3]</sup>. One is based on Hertz theory associating the mass with the contact duration extracted from time-domain signal and the loose part contact radius. The basic assumption are Lamb wave model and complete elastic impact. It utilizes the momentum and a given range of the contact radius to analyze the detected signals. The momentum is figured out by calculating the contact force and the duration time of the contact. Then the algorithm ejects the impossible couple of the force and the contact duration time according to the range of the contact radius. The mass can be estimated by calculate the average values estimated in the possible couples and correction element. The other one uses the Fourier spectrum to estimate the mass. As a heavier

loose part leads to generate more energy with lower frequency, it divided the Fourier transform spectrum into two sectors which can be called as high frequency sector and low frequency sector with a proper frequency determined by experiment. Then the ratio of the high and low sectors for a series of masses are calculated and then used to fit a curve according to the related mass values. The mass estimation is realized by using the ratio of the signal to be estimated and the fitting curve.

In practical environment, the complex vibration wave generated by the loose part consists of bending waves with characteristic of dispersion. During the propagation, the flexible wave is often contaminated by its reflection caused by the structure of the pressure vessel and scattering, which effects the wave form and the accuracy of the evaluated contact duration. The distribution of amplitude in the frequency-domain is effected by the mass but the ratio is not sensitive enough to the varying of mass value and no perfect mapping relation exists between the mass and the Fourier frequency ratio of two sectors.

Based on the relation between the mass value and the frequency distribution of an impact signal, a method of loose part mass estimation using multiple regression is proposed. Marginal Hilbert spectrum, which is chosen to express the feature of the mass, is introduced in the second part of the paper. The multiple regression model for mass estimation is illustrated in the third section which can be built by using the mass features. The fourth part describes the design of the simulated experiment. Experiment data are analyzed in the fifth part and the conclusion is given in sixth section.

## 2. Marginal Hilbert Spectrum

Hilbert-Huang Transform (HHT) is one of the time-frequency analysis method invented by N. E. Huang<sup>[4]</sup> in 1998. The instantaneous frequency defined by traditional Hilbert transform is meaningful only when the signal is narrow-band. Huang proposed the Empirical Mode Decomposition (EMD) to change the non-stationary signal into a number of Intrinsic Mode Functions (IMF) of limited number before the Hilbert transform. Then Hilbert transform is applied to the IMF and finally gets the time-frequency spectrum of the original signal.

## 2.1 Empirical Mode Decomposition

EMD assumes that any time-series consists of a series of different and non-sine intrinsic mode functions. From high frequency to low frequency, it decomposes the time-series into several IMF which follow two conditions: (1) in the whole data set, the number of extreme and zero crossings must either equal or differ at most by one; (2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

Firstly, EMD calculates two cubic spline envelopes for the maxima and minima extreme respectively. Assumes that the mean series of the envelopes is  $m(t)$  and the margin of the signal and the mean is  $c(t)$ , then the first extracting of IMF can be expressed as

$$c_1(t) = x(t) - m_1(t) \quad (1)$$

Usually,  $c_1(t)$  gained above can not satisfy the constrain conditions of IMF. Then the calculating continues by replacing the original signal with  $c_1(t)$  until the  $k$  th margin satisfies IMF conditions.

$c_{1k}(t)$  is the first IMF.

$$c_{1k}(t) = c_{1(k-1)}(t) - m_{1(k-1)}(t) \quad (2)$$

After being processed by EMD, original signal  $x(t)$  can be expressed as the sum of the IMF  $c_i(t)$  and a residual component  $r_n(t)$  obtained by repeating the above algorithm.

$$x(t) = \sum_{i=1}^n c_i(t) + r_n(t) \quad (3)$$

## 2.2 Marginal Hilbert Spectrum

Applying Hilbert transform to the IMF<sup>[5,6]</sup>,  $h_i(t)$  is used to represent the Hilbert spectrum of the  $i$  th IMF

$$h_i(t) = \frac{1}{\pi} \int \frac{c_i(\tau)}{t - \tau} d\tau \quad (4)$$

Then the analytic signal  $C_i(t)$  of  $c_i(t)$  can be

$$C_i(t) = c_i(t) + jh_i(t) \quad (5)$$

And  $c_i(t)$  can be expressed as

$$c_i(t) = a_i(t)e^{j\theta_i(t)} \quad (6)$$

Where, the amplitude  $a_i(t) = \sqrt{c_i^2(t) + h_i^2(t)}$  and the phase function  $\theta_i(t) = \arctan[h_i(t)/c_i(t)]$ . The instantaneous frequency of the  $i$  th IMF is

$$\omega_i(t) = \frac{d\theta_i(t)}{dt} \quad (7)$$

Hilbert time-frequency spectrum (Hilbert spectrum) of the signal is obtained by omitting the residual component and using instantaneous frequency and amplitude.

$$H(\omega, t) = \text{Re} \sum_{i=1}^n a_i(t) e^{j \int \omega_i(t) dt} \quad (8)$$

Where Re stands for extracting the real part of the expression.  $H(\omega, t)$  represents the distribution of the instantaneous amplitude on the time-frequency domain. Marginal Hilbert Spectrum is derived via

time integration of Hilbert spectrum.

$$H(\omega) = \int H(\omega, t) dt \quad (9)$$

## 2.3 Simulated Signal Test

MHS means the sum of the amplitude of the signal at every frequency point and represents the distribution of the signal energy in frequency domain. Based on the instantaneous frequency, it describes the local characteristics of signal more precisely than Fourier spectrum does. In analyzing the non-stationary signals with feature of time-varying frequency, MHS represents the real property of the vibration almost exactly.

Two numerical simulated signals, including a periodic signal and an aperiodic signal, are presented here:

$$1) x_1(t) = \sin(60\pi t) + \sin(160\pi t)$$

$$2) x_2(t) = [\sin(60\pi t) + \sin(160\pi t)] \exp(-8t)$$

Here the time interval is 0.001s, signal length is 0.5s, the frequencies are 30Hz and 80Hz respectively. Fourier spectrum and MHS and the original two signals are shown in Fig.1 and Fig.2. For the purpose of convenient comparison, the amplitudes in the following figures are normalized.

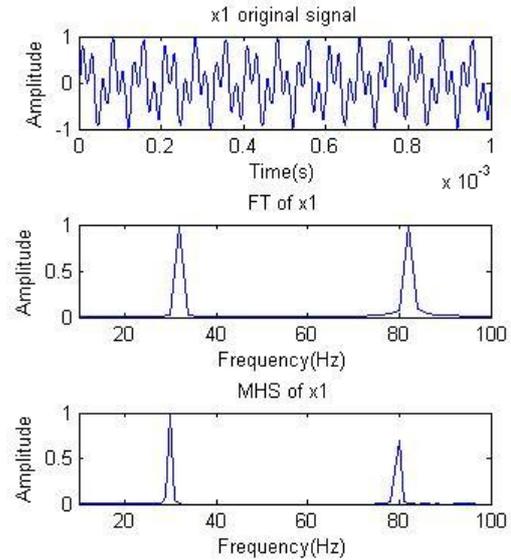


Fig.1 Original signal of  $x_1$  and its FT and MHS.

Fig.1 and Fig.2 shows that as to a periodic signal, the difference between the performances of FT and MHS is relatively small but frequency expression of the FT spectrum happens to have a deviation from the real frequencies. And the deviation would be distinct when the length of the signal is small. As to the decaying exponential aperiodic signal, MHS is capable to describe the exact frequency with good performance in frequency concentricity. So it can be investigated that MHS acts to have better resolution and better expression of the real frequency description.

Loose part impact signal is one of the time-varying aperiodic non-stationary signals. Derived from Hilbert time-frequency spectrum which contains information of the distribution of instantaneous frequencies, MHS expresses the exact amplitude scattered on the frequency axis of the impact signal. According to the applicability of MHS to the impact signal and the relation between the mass and its frequency content, MHS is determined to be able to feature the masses of different loose parts in this paper.

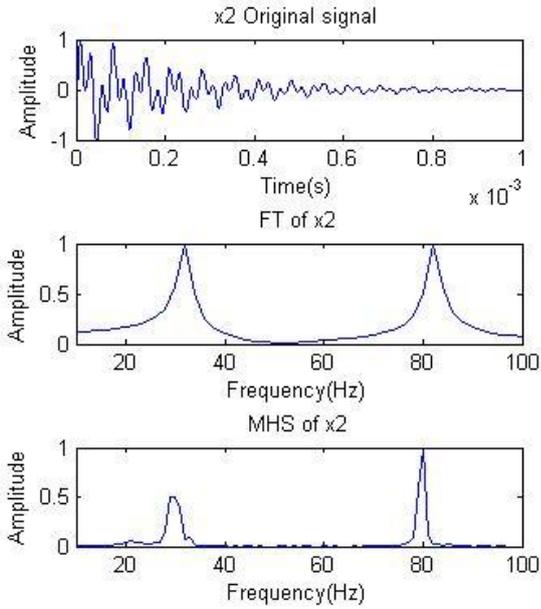


Fig.2 Original signal of  $x_2$  and its FT and MHS.

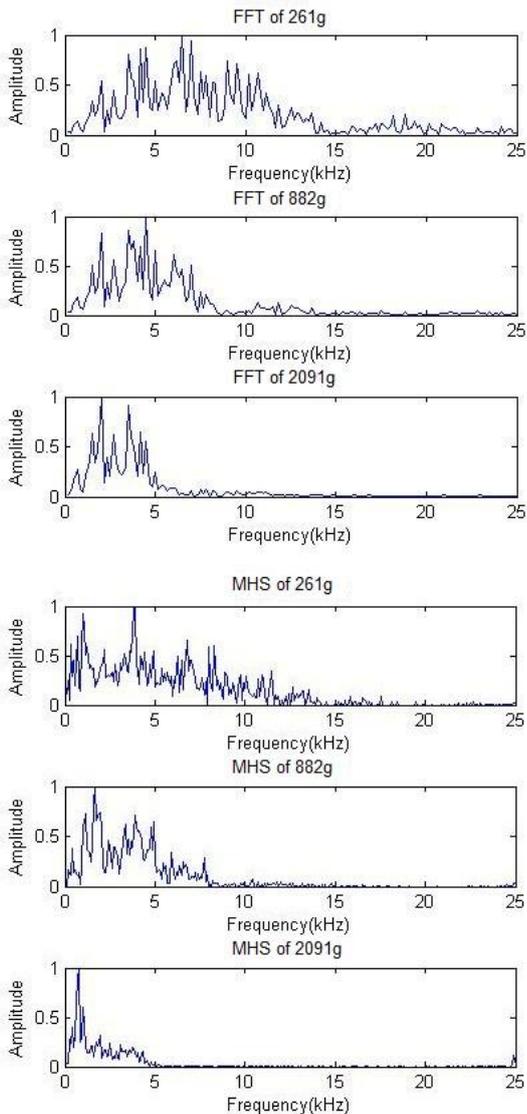


Fig. 3 The Fourier spectrum of different mass signals. As shown in Fig. 3 marginal Hilbert spectrums of different masses has similar trend as the Fourier spectrum.

### 3. Multiple regression model for mass estimation

Multiple regression is used to study the relationship between several variables. The defined regression model can then be applied to the unknown data to perform further research. Based on the relation between the mass and its frequency content, multiple regression is suitable to be adopted for mass estimation by defining the feature of MHS as the argument and the mass value as the induced variable.

Considering the special environment in a nuclear power plant, the number of the practical samples of loose part impacts are rare and the number of the simulated impacts when installing the LPMS are fairly limited. Support Vector Regression (SVR) which is capable to deal with the data set of small number of samples by searching the optimal solution between the learning ability of the limited samples and the complexity of the model to be trained<sup>[7]</sup>, is chosen to be the multiple regression algorithm for mass estimation.

Assuming that the masses of loose parts is expressed by the data set of  $\{(h_i, m_i) | i=1, \dots, n\}$ , in which  $h$  represents the feature of MHS and  $m$  means mass value of corresponding loose part. Under SVR model, the relation between argument  $h$  and dependent variable  $m$  can then be defined as

$$m = f(h) \tag{10}$$

$$f(h) = \langle \omega, \phi(h) \rangle + b$$

Here  $\omega$  is the normal vector of the hyper plane,  $b$  is the bias of the hyper plane,  $\phi(\cdot)$  maps the eigen vector to the high-dimensional feature space, which changes the nonlinear fitting process in the current space into linear fitting process in a new space with higher dimension. Considering the construct risk of model, slack variable  $\xi$ , cost parameter  $c$  and  $\varepsilon$ , which is the maximum acceptable error of the regression, are added into the model. Optimal  $\omega$  and  $b$  in  $f(h)$  can be obtained by calculating the optimal solution of the problem below

$$\min_{\omega, b, \xi, \xi_i^*} \frac{1}{2} \omega^T \omega + C \sum_{i=1}^n \xi_i + \sum_{i=1}^n \xi_i^* \tag{11}$$

Subject to

$$\begin{aligned} (\omega^T \phi(h_i) + b) - m_i &\leq \varepsilon + \xi_i, \\ m_i - (\omega^T \phi(h_i) + b) &\leq \varepsilon + \xi_i^*, \\ \xi_i, \xi_i^* &\geq 0, i=1, \dots, n, \varepsilon \geq 0 \end{aligned}$$

By using Lagrange multiplier  $\alpha$ , the dual problem of (11) is

$$\begin{aligned} \min_{\alpha, \alpha^*} \quad &\frac{1}{2} (\alpha - \alpha^*)^T Q (\alpha - \alpha^*) + \\ &\varepsilon \sum_{i=1}^n (\alpha_i + \alpha_i^*) + \sum_{i=1}^n (\alpha_i - \alpha_i^*) \end{aligned} \tag{12}$$

Subject to

$$\begin{aligned} e^T (\alpha - \alpha^*) &= 0, \\ 0 \leq \alpha_i, \alpha_i^* &\leq C, i=1, \dots, n \end{aligned}$$

Where  $Q_{ij} = K(h_i, h_j) \equiv \phi(h_i)^T \phi(h_j)$ ,  $K$  means kernel function which can replace the high-dimensional inner product in solving the constrained optimal problem by calculating the inner product of current vector in high-dimensional space and  $\phi(h)$  means the mapped  $h$  in high-dimensional space. Mapping to the space with higher dimension will not change the complexity of the algorithm but change the way to calculate the inner product only and will not affect the popularization either<sup>[8]</sup>. Kernel functions which are widely used are Polynomial kernel, Sigmoid kernel and Gaussian Radius Basis Function (RBF) kernel. Because of its unlimited dimensions, RBF kernel is chosen to solve the nonlinear problem in this paper. The expression of RBF kernel is

$$K(h_i, h_j) = \exp\left[-\frac{|h_i - h_j|^2}{\sigma^2}\right] \quad (13)$$

Where,  $\sigma$  is the radius of the hyper-balls around the samples. It means the sensitivity of the model while facing a varying input. The approximate form of mass model can be obtained by solving the symmetry problem

$$\sum_{i=1}^n (-\alpha_i + \alpha_i^*) K(h_i, h_j) = 0 \quad (14)$$

Then the mass estimation relation model based on the feature of MHS and multiple regression is

$$m = \sum_{i=1}^n (-\alpha_i + \alpha_i^*) K(h_i, h_j) \quad (15)$$

The processing steps of the method stated above can be displayed as the figure below:

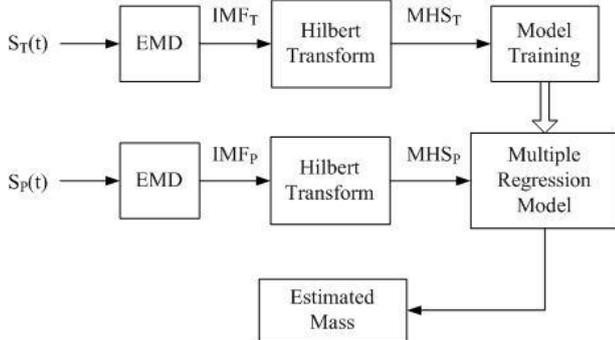


Fig.4 Processing steps of the mass estimation method.

Where,  $S_T(t)$  and  $S_P(t)$  represents the time-series signal used to train the multiple regression model and the signals generated by the balls with unknown mass which waits to be predicted, respectively. And the variables with same subscript mean the corresponding IMF series and MHS feature series.

4. Verification Experiment Design

Simulated experiment on flat plate is designed to verify the feasibility of the proposed mass estimation method. Steel plate (45#) is used to imitate the wall of the pressure vessel of the reactor in nuclear power plant. For the purpose of minimizing the influence of the environmental vibration, four elastic dampers are used to support the steel plate at every corner. Steel balls of different diameters are used to simulate the loose parts with different masses. The sampling frequency of the synchronous data collecting card (FCFR-USB2066, 16 Bits) used in the experiment is 200kHz. During the experiment, an acceleration sensor is used to detect the simulated loose part impact signals which are generated by the dropping different balls on two positions named P1 and P2 on the plate. Fig. 2 shows the schema of the simulated experiment.

The diameters of the steel balls are 40mm, 50mm, 60mm, 70mm, 80mm, 90mm respectively. Every ball impacts each point for five times. After all of the signals have been decomposed by using EMD, MHS can be obtained by applying Hilbert transform to the gained IMF. Mass estimation model, which is trained by utilizing SVR and the MHS of the signal from P1 as the signal features, can then be used to predict the mass values according to the MHS from P2.

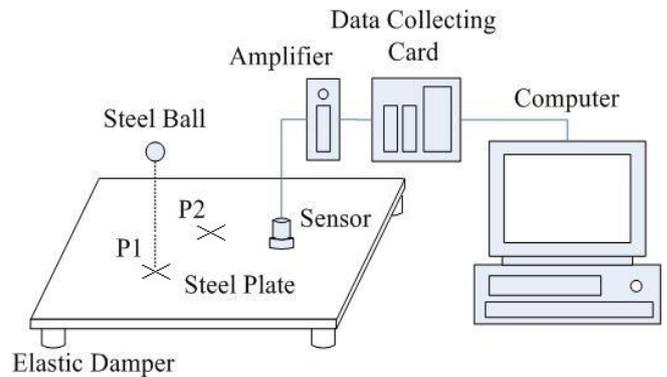


Fig. 5 The schema of the experiment and the impact locations in simulated experiment.

5. Experiment data analysis

There are six kinds of the steel balls with different mass which impact each of the two positions on the steel plate for 5 times. Data gained from P1 are used to get the corresponding MHS to be the features vectors of the masses for training the multiple regression model based on SVR. Mass estimation is then carried out by putting the feature vectors of the signals from P2 into the regression model. The results of mass estimation are shown in Table. 1. Here,  $Avg.M$  means the average of the estimated mass,  $e_{max}$  and  $e_{min}$  means the maximum and minimum relative error of the estimated values respectively,  $Avg.e$  means the average of relative errors.

Results of the masses estimated for different steel balls of diameters

Diameter (mm)	Mass (g)	Estimated Mass (g)					Avg.M (g)	$e_{max}$ (%)	$e_{min}$ (%)	Avg.e (%)
		1	2	3	4	5				
40	261	302.14	303.22	268.53	273.45	284.21	286.83	16.00	2.73	9.53
50	510	446.96	395.07	548.69	470.74	393.96	465.37	22.83	7.48	14.63
60	882	1052.31	973.92	815.12	722.41	745.62	890.94	19.28	7.60	14.18
70	1400	1239.87	1628.43	1163.35	1139.43	1348.87	1292.77	18.61	3.65	13.38
80	2091	2298.96	2331.43	2458.58	2332.91	2277.08	2355.47	17.58	8.90	11.90
90	2997	2811.98	2766.40	2539.58	2784.97	2613.62	2725.73	15.26	6.17	9.80
							<b>Avg.e</b>	18.26	6.09	12.24

Table. 1 The results and the relative errors of the estimated masses.

from 40mm to 90mm with their corresponding masses from 261g to 2997g in Table. 1 show that the maximum relative error between the real mass value and the estimated mass value is about 23%, minimum relative error is 2.73% and the max average relative error is 14.63%. The errors behaves stably in all of the mass values. The mean of all the errors is 12.24%. The errors are fully acceptable in practical using, and the method of using MHS and multiple regression is therefore feasible.

## 6. Conclusion

In this paper, a new method based on the features of the marginal Hilbert spectrum and the multiple regression by using support vector regression for loose part mass estimation is proposed. MHS, which acts better in expressing the real frequency distribution of the time-series signal than FT, is capable to describe the mass features of the impact signals. Data analysis of the simulated impact experiment shows that the error of the estimated mass is stable for different masses, the average error is 13.79% which is acceptable in engineering application. The experiment verifies that the mass feature based on MHS is feasible and the mass estimation method based on multiple regression is reliable.

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