Evaluation and Modeling of Geometric Form Error for Precision Assembly Analysis

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Abstract: The surface geometric form of any machined feature always deviates from its nominal design to some degree. Extensive researches have been carried out to show that surface geometric deviations, such as roughness and form errors, can impact the assembly accuracy and eventually the performance of precision mechanical system. Surface geometric form error was focused on in this paper. In order to evaluate the assemblability of components which relates to geometric form error, an integrated method combining statistics and concept of entropy was proposed to evaluate geometric form error. The method innovatively adopts entropy to evaluate the uniformity of geometric form error. Simulation results show that the uniformity of geometric form error can be appropriately represented by one parameter. On the basis of obtained evaluation results, mode shape decomposition method was adopted and improved to be more accurate and simpler to model measured geometric form error. The results indicate that the rebuilt geometric form error consists well with the measured one. Finally, we come to conclusions that valuable information of geometric form error can be derived to lay a foundation for assembly analysis through entropy evaluation and mode shape decomposition modeling methods.

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1. Introduction

The geometric form of any manufactured feature always deviates from its nominal design to some degree, owing to the random and/or systematic errors [1]. Generally, identifying geometric form errors needs comparing the design specifications with the geometry form rebuilt from a finite number of measured points on the manufactured surface through a geometric filtering procedure [2]. The skin model [3] is a measurable surface, which must be filtered (parameterized) in order to take into account the errors. The filtering can be made by several methods. Most of them use periodic functions that can build simple to complex shapes. The Fourier series is the first used in the decomposition method in order to define periods on circular [4] and linear shapes. Two dimensional Fourier is used by Capello and Semeraro [5, 6] to define the form parameters of rectangular shapes. Huang and Ceglarek [7] developed a mode-based method for form error decomposition and characterization by using discrete cosine transform. A Tchebychef Fourier Series model is used by Henke *et al*. [8] in order to describe the forms of a cylinder and identify specific types of error shapes. Serge Samper [9] proposed a new way to define form error parameters based on the eigen-shapes of natural vibrations of surfaces.

Neville [10] studied the effects of surface roughness of workpiece and datum on the locating precision of multi-station manufacturing processes using a mechanical system. The results show that geometric form is extremely important for many multi-station applications. Samper [11] modeled the 2D and 3D assemblies taking into account geometric form errors of planar surfaces. It can be easily shown that parts satisfying the same form tolerance can have very different profiles, which may result in substantial differences in assembly accuracy. As precision requirement becomes higher, geometric tolerances to control geometric form errors must be considered.

The objective of this paper is to develop evaluating and modeling methodologies for geometric form error. Here the geometric form error, defined as the difference of actual and nominal surface, has sufficient smoothness such that the high spatial frequency components are small and can be ignored.

2. Evaluation of geometric form error

2.1 Theoretical Background

At present, traditional 2D surface topography parameters are still used to evaluate surface topography in engineering. However, they excessively focus on information in height direction and ignore information in horizontal direction, which cannot reflect real surface topography and results in lost of extensive valuable information. The geometric form errors shown in Fig. 1 are identical according to traditional evaluation methods, but their surface topography and functional performance are quite different. Traditional evaluation method can lead to great inaccurate evaluation results.

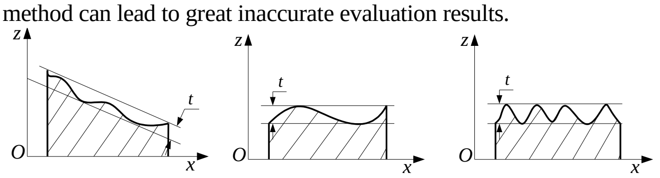


Fig. 1 Geometric form error

Entropy is a metric of disorder, unbalance, uncertainty and non-uniform. In information theory, information entropy $H(X)$ is a function of the distribution of random variable X, so it is also called entropy function. Probability p is generally used as a fundamental quantity to describe information, in which three basic cases exist: p=1 for affirmed case, p=0 for denied case and 0<p<1 for indeterminate case [12]. If we write probability distribution $p(x_i)$ i=1, 2, …, n as p_1, p_2, …, p_n, then entropy function once again can be written as the function $H(p)$ of probability vector $\mathbf{P}=(p_1, p_2, \ldots, p_n)$.

$$H(X) = -\sum_{i=1}^{n} p_i \log_2 p_i = H(p_1, p_2, \cdots p_n) = H(\mathbf{p}) \tag{1}$$

The extremum of entropy function corresponding to Eq. 1 is

$$H_\propto = -\sum_{i=1}^{n} \frac{1}{n} \log_2 \frac{1}{n} = \log_2 n \tag{2}$$

Geometric form error is generally uncertain and non-uniform. Its uniformity directly affects the contact performance, contact stress, and stress field in components, which in turn impacts assembly accuracy and system performance. Therefore, evaluating uniformity of geometric form error can approximately predict its impact and provide basis and guide for improvement of assembly accuracy and system performance.

2.2 Evaluation process of geometric form error

Assume that the surface data matrix **A** of a planar surface is

$$\mathbf{A} = \begin{bmatrix} x_1, y_1, z_1 \\ x_2, y_2, z_2 \\ \vdots \\ x_n, y_n, z_n \end{bmatrix} \tag{3}$$

Where n is the number of measurement points on the surface, x_i, y_i, z_i are the coordinate values of the ith measurement point, respectively.

As mentioned above, the coordinate value Z at different measurement points can be viewed as a random variable. One of the sample observation values of Z, that is the measurement data, is z_1, z_2, …, z_n which can be written $\mathbf{Z}=[z_1, z_2, \ldots, z_n]$.

The sample mean of Z is

$$E(Z) = \mu = \frac{1}{n} \sum_{i=1}^{n} z_i \tag{4}$$

And the sample variance of Z is

$$D(Z) = \sigma^2 = \frac{1}{n} \sum_{i=1}^{n} z_i^2 - \mu^2 \tag{5}$$

μ and σ^2 reflect the distribution of geometric form error. μ reflects the average deviation of the measurement points to the datum surface. In other words, it reflects the average translational deviation of the best fitted ideal surface to nominal surface. Variance σ^2 approximately evaluates the rotating deviation extent of real surface to datum surface. However, neither mean μ nor variance σ^2 evaluates the uniformity of geometric form error. Therefore, entropy was introduced into the evaluation of geometric form error.

Due to nonnegative requirement of independent variable in entropy function, a coordinate system is established taking the lowest position of the surface as zero point of z axis, as shown in Fig. 2, in which real surface and average surface with a height of mean μ are given. Taking the average surface as the center, two assistant surfaces parallel to each other on its both sides with an equal distance to it are established. The distance between the two assistant surfaces is t/2, where t is the flatness tolerance. Fig. 2 (a) is the actual 3D case; and (b) is the corresponding 2D case by rewriting surface height values in vector form.

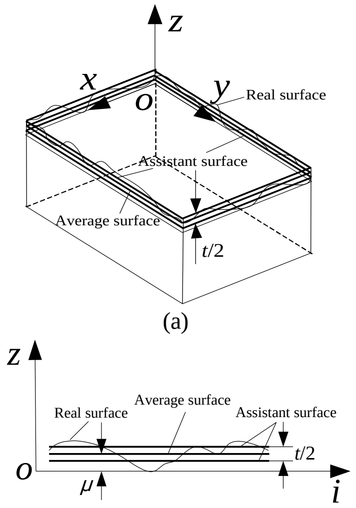


Fig. 2 Definition of surfaces

The evaluation process is as follows:

Step 1: Measurement points locating between the assistant surfaces are viewed as points meeting uniformity requirements, and the number of them is m. First normalization of the z axis coordinate value of these points is carried out to make the sum of them be 1, and then their entropy value is calculated.

$$H = -\sum_{i=1}^{m} z_i' \log_2 z_i' \tag{6}$$

The corresponding entropy extremum is

$$H_\propto = -\sum_{i=1}^{m} \frac{1}{m} \log_2 (\frac{1}{m}) \tag{7}$$

We define standard uniformity coefficient as

$$f_s = H / H_\propto \tag{8}$$

According to property of entropy, the range of standard uniformity coefficient is (0, 1).

Step 2: Normalization of the z axis coordinate value of all measurement points on the surface is made. Overall entropy value H_1 and corresponding entropy extremum $H_{1\propto}$ of these measurement points are calculated. So the uniformity coefficient is $f_1=H_1/H_{1\propto}$. Here we write $\mathbf{F}_1=[f_1]$.

If $f_1 \geq f_s$, the calculation terminates; otherwise we hold that overall geometric form error cannot meet uniformity requirement (defined by assembly), and next step calculation is needed.

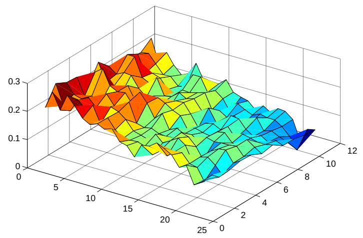
Step 3: The surface is divided into four identical parts for the first time, $L=1$, and each part includes q measurement points. Entropies H_{21}, H_{22}, H_{23} and H_{24} of the four parts and their common entropy extremum $H_{2\propto}$ are calculated. So the uniformity coefficients are $f_{21}=H_{21}/H_{2\propto}$, $f_{22}=H_{22}/H_{2\propto}$, $f_{23}=H_{23}/H_{2\propto}$ and $f_{24}=H_{24}/H_{2\propto}$, respectively. We write them in a vector $\mathbf{F}_2= [f_{21}, f_{22}, f_{23}, f_{24}]$. Then each of these four coefficients is compared with f_s. If they are all larger than f_s, the calculation terminates. Otherwise, the geometric form errors of at least one of the four parts cannot meet uniformity requirements, next step calculation is needed.

Step 4: Second division is implemented to divide each part in Step 3 into four parts, L=2. Then the same calculation procedure is carried out till the uniformity coefficient of each part is larger than the standard uniformity coefficient f_s.

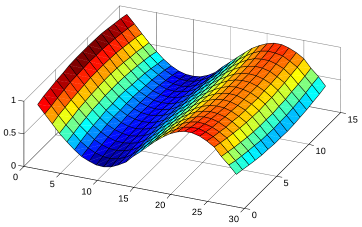
The final division times L above is used to evaluate the uniformity of overall geometric form error, then we call geometric form error has uniformity of L level. The larger is L, the worse uniformity of geometric form error is.

2.3 Evaluation process of geometric form error

We use two simulated geometric form errors of planar surfaces, as shown in Fig. 3, to verify the proposed evaluation methodology. The evaluation results corresponding to surfaces in Fig.3 (a) and (b) are shown in Fig. 4.

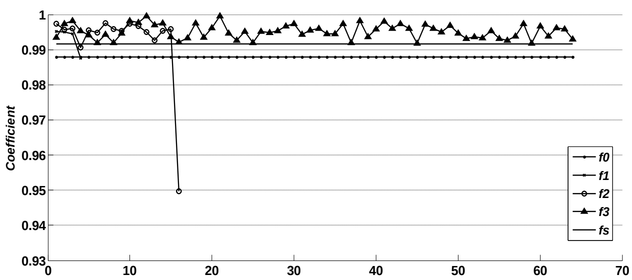


(a)

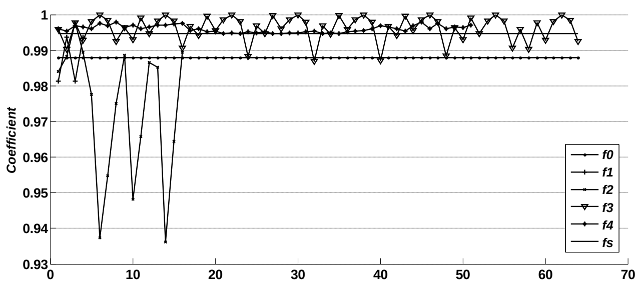


(b)

Fig. 3 Simulated planar surfaces



(a)



(b)

Fig. 4 Uniformity coefficients

Fig. 4 shows that by three times of division the uniformity coefficient of each small part of surface in Fig. 3(a) is larger than f_s, and by four times of division the uniformity coefficient of each small part of surface in Fig. 3 (b) is larger than f_s. Then we call the planar surfaces have a uniformity of level 3 and level 4, respectively. It is very important to note that the proposed method must be applied to a surface with continuity and the number of measurement points must be large enough. These two conditions can be easily met in precision mechanical system.

3. Geometric form error modeling

On the basis of the evaluation results, we use mode shape decomposition method, a geometric form error modeling method proposed by Formosa and Samper [13], to model geometric form error to provide exact guide for assembly.

3.1 Decomposition of form errors

Geometric form error is usually represented as a combination of a series of mode components. The orthogonality of these modes explained below ensures the independency of their coefficients, which is a fundamental prerequisite for statistic geometric tolerance analysis [14]. This is the main reason that mode shapes are used to model geometric form error.

Geometric form error can be represented by a measured displacement vector $\mathbf{V}$, one column of data matrix $\mathbf{A}$.

$$\mathbf{V}= [z_1, z_2, \ldots, z_n] \tag{9}$$

Now we rebuild geometric form error as a linear combination of mode shapes (Eigen shapes) which are orthogonal column vectors whose linear combination can reconstruct some initial data matrices. According to knowledge of linear algebra, a random vector $\mathbf{V}$ can be viewed as a vector in vector space S ($S \subset \mathbb{R}^n$) [15]. To obtain exact linear representation of $\mathbf{V}$, an orthogonal basis of S is needed. If $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n$ is an orthogonal basis of S, then $\mathbf{V}$ can be represented as

$$\mathbf{V} = \lambda_1\mathbf{e}_1 + \lambda_2\mathbf{e}_2 + \cdots + \lambda_n\mathbf{e}_n \tag{10}$$

Where $\mathbf{e}_i=[e_{i1}, e_{i2}, \ldots, e_{in}]^T$, and λ_i (i=1, …, n) are coefficients and can be obtained by inner product with $\mathbf{V}$.

$$\lambda_i = \mathbf{V}^T\mathbf{e}_i = <\mathbf{V}, \mathbf{e}_i> \tag{11}$$

So the geometric form error vector $\mathbf{V}_1$ rebuilt by vector set $\mathbf{e}_i$ is

$$\mathbf{V}_1 = \sum_{i=1}^{n}\lambda_i\mathbf{e}_i = \mathbf{E\lambda} \tag{12}$$

Where $\mathbf{E}$ is a matrix comprised of eigenvector $\mathbf{e}_i$, and $\boldsymbol{\lambda}$ is the vector comprised of mode coefficient λ_i.

Therefore, the residual vector ε is the difference between form error vector $\mathbf{V}$ and the constructed vector $\mathbf{V}_1$.

$$\boldsymbol{\varepsilon} = \mathbf{V} - \mathbf{V}_1 = \mathbf{V} - \sum_{i=1}^{n}\lambda_i\mathbf{e}_i = \mathbf{V} - \mathbf{E}\boldsymbol{\lambda} \tag{13}$$

Therefore, a set of m nominally identical surfaces machined under identical conditions is densely sampled at n locations, resulting in a data matrix, $\mathbf{D}$, of dimensions n×m. We can write

$$\mathbf{D}=\mathbf{BC} \tag{14}$$

Where $\mathbf{B}$ is an n×n matrix of orthogonal basis, and $\mathbf{C}$ is an n×m matrix of surface-specific coefficients appropriately weighting the contributions of various factors in $\mathbf{B}$ for each surface measured.

Each column of $\mathbf{D}$ corresponds to geometric form error of an individual surface, and each column of $\mathbf{B}$ corresponds to an eigen shape corresponding to the deviation (from nominal form) characteristic of an independent process-related factor. In practice, however, it is difficult and unnecessary to include n (column) eigen shapes in **B**. It is reasonable to use an appropriate subset k eigen shapes in **B** to rebuild geometric form error in **D**, then Eq. 14 can be written as

$$\mathbf{D} \approx \mathbf{D}_1 = \mathbf{B}_k \mathbf{C}_k \tag{15}$$

where $\mathbf{B}_k$ is a $(n \times k)$ submatrix of **B**, and $\mathbf{C}_k$ is a $(k \times m)$ submatrix of **C**.

Mode shape has orthogonality about mass matrix **M** and stiffness matrix **K** [16], that is, in a multi-degree of freedom discrete of continuous system mode shapes $\mathbf{U}_i$ and $\mathbf{U}_j$ corresponding to two different frequencies ω_i and ω_j have relationship below

$$\mathbf{U}_i^{\mathrm{T}} \mathbf{M} \mathbf{U}_j = 0 \qquad (i \neq j) \tag{16}$$

The mass matrix of the system is

$$\mathbf{M} = \begin{bmatrix} m_1 & & & 0 \\ & m_2 & & \\ & & \ddots & \\ 0 & & & m_N \end{bmatrix} \tag{17}$$

So Eq. 16 can be written as

$$\sum_{k=1}^{N} m_k U_{ki} U_{kj} = 0 \tag{18}$$

Here the mass distribution is uniform, that is, $m_1 = m_2 = \ldots = m_N$, so

$$\sum_{k=1}^{N} U_{ki} U_{kj} = 0 \tag{19}$$

To use the mode shape in the modeling of form error, we carry out unitization to vector $\mathbf{U}_i$ and obtain

$$\mathbf{e}_i = \frac{\mathbf{U}_i}{\sqrt{U_{i1}^2 + U_{i2}^2 + \cdots U_{iN}^2}} \tag{20}$$

From Eq. 19 and Eq. 20 we know that mode shapes $\mathbf{e}_i$ and $\mathbf{e}_j$ are orthogonal.

The geometric form error modal shape decomposition concept will help the identification and separation of different error pattern sources, for example, in a final product rigid modal shapes represent datum of locator induced part position/orientation deviation, while the deformation modal shapes reveal the error caused by parts manufacturing, such as springback.

3.1 Modal basis construction

Several kinds of commercial FEA software are available to construct modal basis. The geometric form error relates only to the displacement of surface nodes. Therefore, SHELL element which usually meshes surface and thin plate is used. For convenience, we will mesh the surface in the modal basis construction process to ensure that measurement program and FE model share the same discrete model.

Generally, free boundary conditions should be used. However, in some cases, fixed boundary conditions enable us to take into account some particularities of geometric form errors [17]. In the case of free boundary, there are six rigid body modes in which the ideal surface has small rigid body displacements that can be used to identify the position of the surface from a coordinate system or another datum feature.

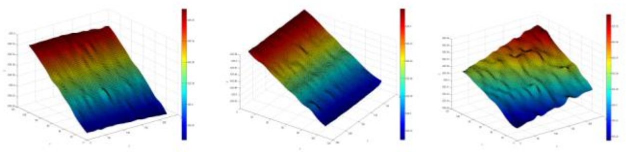
The displacement of each node in one mode shape is written in a vector according to node number. The rigid body displacement vectors are not orthogonal with other displacement vectors, so they, as well as in-plane displacement vectors, will be excluded from the approximate orthonormal basis. Although the rigid body displacement vectors are not considered in the orthonormal basis, we can still project the geometric form error onto them to identify the overall trend of geometric form error.

4. Case Study

Experiments are carried out to get surface geometric form error data to rebuild. Experimental conditions are: machining tool is MM7120A, workpiece material is 45 steel, cutting tool is grinding wheel with dimension of Φ250×25×Φ75mm, and machining parameters are rotary speed of grinding wheel of 15000rpm and vertical feeding of 0.002mm.

We use Croma CMM, a precision coordinate measuring machine, to measure 3 planar surfaces in one single set-up. Common CMM measurement errors, such as probe diameter compensation error, are ignored due to its little ratio to the relatively large manufacturing error. On each surface totally 253 data points are measured.

Fig. 5 shows the measured surface topography of three grinding planes in the same batch machined by the same machine tool under same machining conditions. The geometric form errors of these parts on the whole are same and micro errors are also similar to each other.



(a) (b) (c)

Fig. 5 Surface topography

We rebuild geometric form error as shown in Fig. 5(c). We first align the measurement coordinate of CMM with the position of aforementioned average surface. Mode coefficients of mode shapes as shown in Fig. 6 can be obtained by Eq. 11. The dominant mode shapes can be identified according to mode coefficients. The overall trend of geometric form error is approximated to the rigid displacement of mode 4 as shown in Fig. 7 corresponding to the largest coefficient.

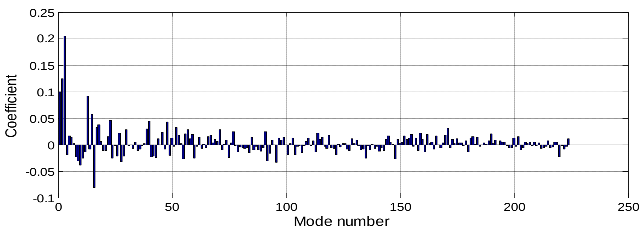


Fig. 6 Mode coefficients

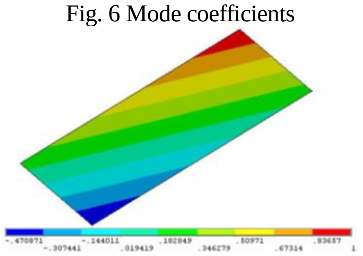


Fig. 7 Rigid displacement indicating the overall trend

The measured geometric form error in vector form and the rebuilt one without any modification are shown in Fig. 8.

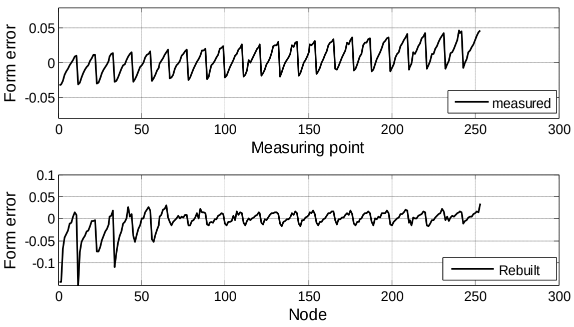


Fig. 8 Measured and rebuilt geometric form error

In Fig. 8, the displacements of the first 64 nodes in the rebuilt form error are larger than and have large deviation to measured form error. The main reason is that the first 64 nodes locate on the boundary of finite element model and their displacements are dominant in mode shapes. Therefore, the rebuilt form error should be modified to represent the measured geometric form error more accurately.

Theoretically, the accuracy of rebuilt surface increases as the number of mode shapes increases. However, in Fig. 6 some mode coefficients are very small, resulting in little contribution to rebuilt surface. Therefore, proper mode shapes should be carefully selected according to mode coefficients to rebuild geometric form error with less mode shapes but without accuracy reduction. Except for the first 4 coefficients, we select the coefficients whose absolute values are larger than the average coefficient absolute value, and 82 coefficients are selected. The mode shapes corresponding to the selected coefficients are called dominant mode shapes. With dominant mode shapes, we rebuild the geometric form error once again. The results are shown in Fig 9.

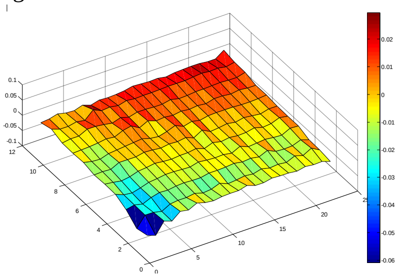


Fig. 9 Rebuilt geometric form error by 82 selected mode shapes

4. Conclusions

The evaluation and modeling methodologies proposed and developed in this paper can provide basis and guide for assembly analysis of mechanical system. Through evaluation of uniformity of geometric form error, we can approximately identify effects of surface geometric form error on assembly accuracy. On the basis of this, mode shape modeling of geometric form error is used to find out the dominant mode shapes, that is, the aspects of geometric form error which have great impacts on assembly accuracy. In addition, geometric form error modeling can be help to identify and separate different error pattern sources.

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