A New Quadrature Demodulation Technique for Self-mixing Interference Signal Analysis

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Abstract: A new, to the best of our knowledge, method for the measurement of the micro-displacement of a remote target based on the laser diode self-mixing interferometry(LDSMI) is presented. Quadrature demodulation technique is introduced to improve the measurement accuracy. Phase modulation of the laser beam is obtained by an electro-optic modulator (EOM) in the external cavity. Detection of the target's displacement can be easily achieved by sampling the interference signal at those times which satisfied certain conditions. The major advantage of the technique is that it does not involve any complicated calculation and insensitive to the sampling error. Experimental results show that the proposed method can effectively improve the displacement measurement accuracy of the LDSMI system to a few nanometers.

Keywords: Self-mixing interferometry, quadrature demodulation, displacement measurement

1. Introduction

As the technology of precision manufacturing advances, it becomes more and more important to measure and control with high accuracy micro-displacement of a machine tool. Recent advances in self-mixing interferometer enable to realize compact, remote control, automatic measuring systems which provide data about static or dynamic behavior of micro-elements with nanometer accuracy. Laser diode self-mixing interferometry(LDSMI) have attracted more and more attention because of the inherent simplicity, compactness and robustness as well as the self-aligning. LDSMI was used to measure displacement with an accuracy of $\lambda/2$ by counting the interference signal peaks. By this method we can only measure the micro-displacement with an accuracy of $\lambda/2$. Phase measurement methods have been widely used in a variety of conventional interferometers because of their high accuracy, hence these methods was also proposed to increase the accuracy beyond $\lambda/2$ for self-mixing interferometer. Injection current modulation is the most common modulation method in which the wavelength is modulated by changing the injection current of a LD[1-3]. Some demodulation techniques have been reported based on current modulation. The main disadvantage of modulating the injection current of a LD is: by varying the injection current, the wavelength of the LD is modulated, but the intensity modulation concurrent with the wavelength modulation leads to measurement errors. In this paper a new method for the measurement of the micro-displacement of a remote target based on LDSMI is presented. Phase modulation is obtained by an electro-optic modulator (EOM) in the external cavity. Phase demodulation is obtained by the quadrature demodulation technique. In this technique, detection of the object's displacement can be easily achieved by sampling the interference signal at those times which satisfied certain conditions. The major advantage of the technique is that it does not involve any complicated calculation and insensitive to the sampling error.

2. Theoretical analysis

2.1 Basic theory of self-mixing interferometry

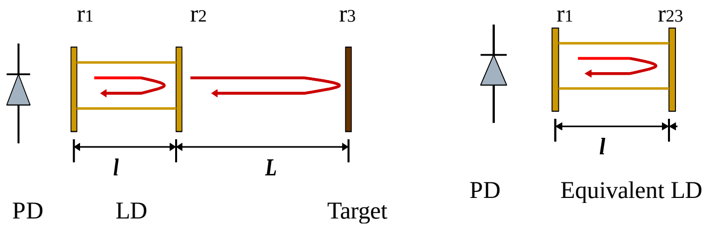


Fig.1 Schematic diagram of the optical feedback effect.

(a) A laser diode with an external cavity; (b) Equivalent model

The basic theories of the self-mixing effect can be explained by two Fabry-Perot cavities shown in Fig. 1(a) and its equivalent Fabry-Perot is represented by Fig. 1(b). Here r_1 and r_2 are the amplitude reflectivity of LD facets, r_3 is the amplitude reflectivity of the external target, l is the length of the laser cavity and L is the length of the external cavity. The laser beam emitted from a LD is reflected from the target, a portion of the laser output back into its laser cavity, and the laser feedback effect occurs. Neglecting the multiple reflections in the external cavity, the frequency of the equivalent cavity υ and the emitted optical power can be expressed as[4]

$$\upsilon_0 - \upsilon = \frac{C}{2\pi t_L}\sin(2\pi\upsilon t + \arctan\alpha) \tag{1}$$

$$I = I_0(1 + m\cos\phi) = I_0\left[1 + m\cos(4\pi\upsilon L / c)\right] \tag{2}$$

Where $I0$ and $\upsilon 0$ are the optical power and the optical frequency without feedback light, α is the linewidth enhancementfactor, and m is the undulation coefficient. The external feedback strength parameter C is defined by

$$C = \frac{t_L}{t_l}\zeta\sqrt{1+\alpha^2} \tag{3}$$

The parameter ζ is the coupling coefficient, and t_L and t_l denote the flight times within the external cavity and the laser cavity. From Eq. (1), we can see when the self-mixing interferometer operates in a week feedback regime (i. e. C is very small), the variation in the laser frequency caused by optical feedback can be neglected. Then the phase of the SMI signal only depends on the external cavity length. The signal peaks appear when the following relationship is satisfied

$$\Delta\phi = \frac{4\pi\upsilon}{c}\cdot\Delta L = 2\pi \Leftrightarrow \Delta L = \frac{\lambda}{2} \tag{4}$$

As a consequence, we can only measure the displacement of the external target with an accuracy of $\lambda/2$. To achieve higher accuracy, quadrature demodulation technique is introduced into LDSMI as below.

2.2 Principle of quadrature demodulation technique

In order to determine the phase φ, an experimentally controlled additive phase $\pi/2\cos(\omega_m t)$ is introduced by EOM situated in the external cavity. Where ω_m is the angular frequency of the modulation signal. Considering that the beam pass through the EOM twice in the external cavity, the modulated interference signal can be written as:

$$\begin{aligned} I(t) &= I_0\left\{1+m\cos\left[\phi+2\psi(t)\right]\right\} \\ &= I_0\left\{1+m\cos\left[\phi+\pi\cos(\omega_m t)\right]\right\} \end{aligned} \tag{5}$$

Expanding Eq. (1) , $I(t)$ can be expressed as:

$$\begin{aligned} I(t) &= I_0\left\{1+m\cos\left[\phi+\pi\cos(\omega_m t)\right]\right\} \\ &= I_0+mI_0\{\cos\phi\cos[\pi\cos(\omega_m t)]-\sin\phi\sin[\pi\cos(\omega_m t)]\} \\ &= I_0+mI_0[\cos\phi E(t)-\sin\phi F(t)] \end{aligned} \tag{6}$$

Where

$$E(t) = \cos[\pi\cos(\omega_m t)] \tag{7}$$

$$F(t) = \sin[\pi\cos(\omega_m t)] \tag{8}$$

In order to achieve the phase φ from the self-mixing interference signal, a special sampling technique is introduced as shown in Fig. 2. Fig.2(a) is the phase modulation signal $2\psi(t)$ introduced by the EOM. In a modulation period, the optical intensity I(t) is sampled at the instant $t_i = \pi i/6\omega_m$, with $i\in[0,11]$ which is represented by S*i* as shown in Fig.2(b). From Fig. 2(c) we can see that at the sampling instant which satisfied $\omega_m t_i = (0, \pi/2, \pi, 3\pi/2)$, E(t) is at extreme point and F(t) is zero whose values are listed in Table. 1.

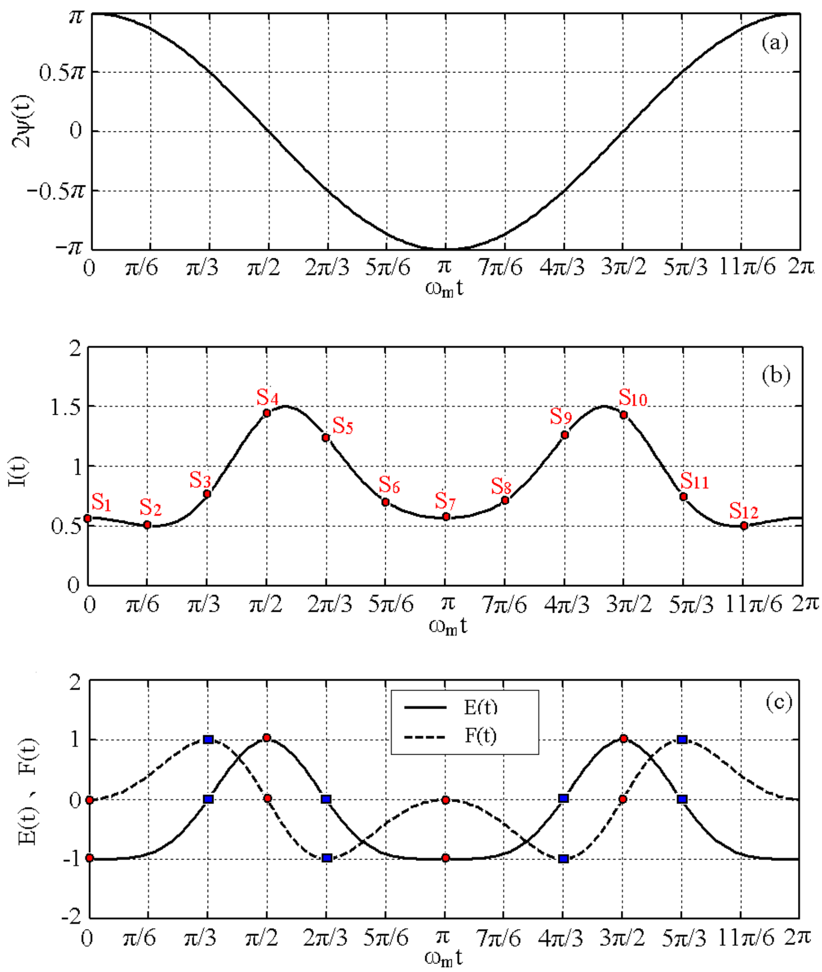


Fig.2 Principle of sampling technique of the quadrature demodulation technique

Table. 1

$\omega_m t$	0	$\pi/2$	π	$3\pi/2$
E(t)	-1	1	-1	1
F(t)	0	0	0	0
I(t)	S_1	S_4	S_7	S_{10}

Since E(t) is at extreme point with the minimum gradient in Table. 1, the variation of E(t) caused by sampling error can be minimized. And the variation of F(t) caused by sampling error can also be eliminated. From Eq.(8) we can see that F(t) is odd-symmetry with respect to $\pi/\omega m$ which can be expressed as:

$$F(t) = -F\left(t + \frac{\pi}{\omega_m}\right) \tag{9}$$

As shown in Talbe. 1, the time sampling interval between S_4 and S_{10}, S_1 and S_7 both are $\pi/\omega m$, the variation of F(t) caused by sampling error can be eliminated by $S_4 + S_{10}$ or $S_1 + S_7$. In order to eliminate the DC component of I(t), linear combinations of S_i in Table. 1 can be formed to have

$$CR = (S_4 + S_{10}) - (S_1 + S_7) = 4mI_0 \cos\phi \tag{10}$$

On the other hand, we can see that at the sampling instant which satisfied $\omega_m t_i = (\pi/3,$ $2\pi/3$, $4\pi/3$, $5\pi/3)$, E(t) is zero and F(t) is at extreme point whose values are listed in Table. 2.

Table. 2

$\omega_m t$	$\pi/3$	$2\pi/3$	$4\pi/3$	$5\pi/3$
E(t)	0	0	0	0
F(t)	1	-1	-1	1
I(t)	S_3	S_5	S_9	S_{11}

Considering that F(t) is at extreme point with the minimum gradient in Table.2, the variation of F(t) caused by sampling error can be minimized. And from Eq.(7) we can see that E(t) is even-symmetry with respect to π/ωm which can be expressed as:

$$E(t) = E(t - \frac{\pi}{\omega_m}) \tag{11}$$

Since the time sampling interval between S3 and S9, S5 and S11 both are π/ωm, the variation of E(t) caused by sampling error can be eliminated by S9-S3 or S5-S11. In order to eliminate the DC component of I(t), linear combinations of S_i in Table. 2 can be formed to have

$$SR = (S_9 - S_3) + (S_5 - S_{11}) = 4mI_0 \sin\phi \tag{12}$$

Then the phase Φ can be calculated using the following relationship:

$$\phi = \arctan(SR / CR) \tag{13}$$

The phase Φ obtained using the demodulation technique proposed above is wrapped within the region of $-\pi$ and $-\pi$. After a phase unwrapping process and following the relationship between the phase Φ and the length of the external cavity, the movement of the external target can be reconstructed.

3. Experimental setup and results

The experimental setup is shown is Fig. 3. It consists of a LD package (operating in single mode) with a photodetector, an aspherical collimating lens, an EOM (New Focus 4002) and the external target. The central wavelength λ_0, maximum output power of the LD are 638 *nm* and 5*mW* respectively. The temperature of both the laser mount and the lens holder, which are embodied in one piece of aluminum alloy is stabilized at ΔT<0. 01°*C*. And the driving current of LD is stabilized at Δi<4μA. The angle between the polarization direction of the laser diode and the electro-optically active axis of EOM is 0^0. Then the EOM can provide pure phase modulation with extremely low amplitude modulation. The external target is fixed on a high-precision commercial PZT (PI, P-841.10) which can obtain displacement accuracy of 0.15nm based on the close-loop design. The voltage signal from the PD is amplified and then digitized with a 200-KHZ, 12-bit analog-to-digital board on a PC bus (National Instrument, NI 6024E).

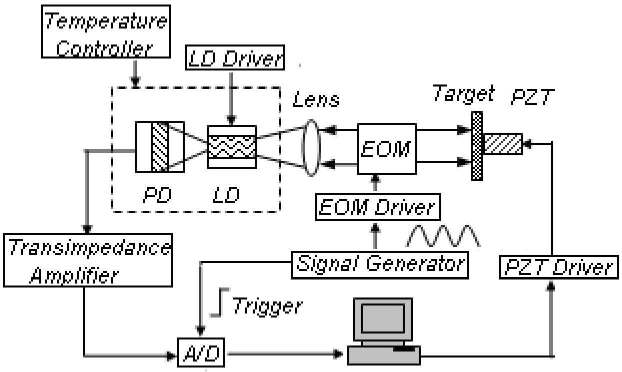
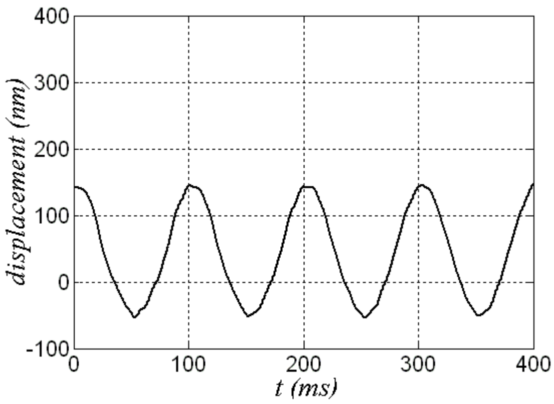


Fig. 3 Experimental setup

Experiments have been done to confirm the validity of the quadrature demodulation technique for self-mixing interference signal analysis. First, the PZT is controlled to move at a sinusoidal form with frequency10HZ and amplitude 200nm (peak to peak). Using the quadrature demodulation technique proposed, the micro-displacement is reconstructed as shown in Fig. 4. The peak-to-peak amplitude of the reconstructed waveform in Fig. 4 are 193.7nm, 198.6nm, 200.4nm and 195.9nm respectively. Then, the PZT is controlled to move at a sinusoidal form with frequency10HZ and amplitude 600nm (peak to peak). The measurement result is shown in Fig. 5 and the peak-to-peak amplitude of the reconstructed waveform are 608.2nm, 605.9nm, 604.4nm and 603.3nm. At last, the PZT is controlled to move at a sinusoidal form with frequency 25HZ and amplitude 2000nm (peak to peak). The measurement result is shown in Fig. 6 and the peak-to-peak amplitude of the reconstructed waveform are 1997.2 nm, 2002nm, 1999.8nm and 1997nm. From above experimental results, we can see that the proposed quadrature demodulation technique can effectively improve the displacement measurement accuracy of the LDSMI system to a few nanometers.



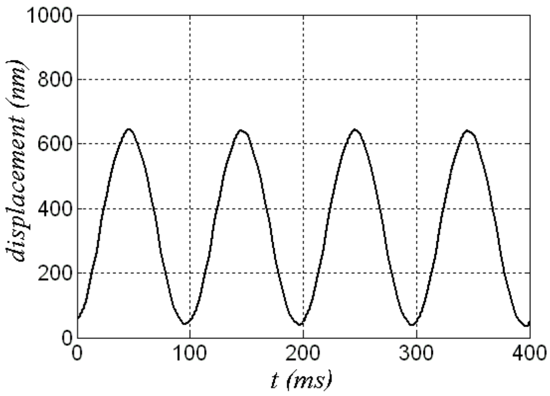


Fig. 4 Measurement result of micro-displacement with with amplitude 200nm (p-p) and frequency10HZ.

Fig. 5 Measurement result of micro-displacement amplitude 600nm (p-p) and frequency10HZ.

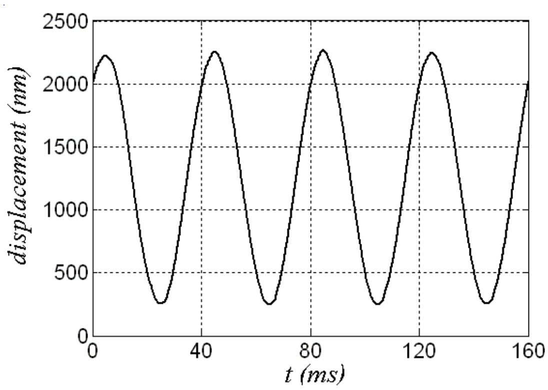


Fig. 6 Measurement result of micro-displacement with amplitude 2000nm (p-p) and frequency 25HZ.

4. Conclusions

Quadrature demodulation technique is first introduced in the self-mixing interference signal analysis. In our experiment, a micro-displacement measurement accuracy of a few nanomters has been obtained. The major advantage of the technique is that it does not involve any complicated calculation and insensitive to the sampling error. Moreover, the simplicity of the demodulation method makes it possible to realize real-time displacement measurement.

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