# Dynamic Specifications by Forward-Looking Controlling

**Carlos Hernández[1], Marc Fischer[2], Rainer Tutsch[2]**

Institute of Production Metrology, Technische Universität Braunschweig
Schleinitzstraße 20, D-38106 Braunschweig, Germany
[1]carlos.hernandez@tu-bs.de, [2]r.tutsch@tu-bs.de , TEL: +49-531-391-7029, FAX: +49-531-391-5837

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*Commonly process control models relay on closed-loop controllers to feedback the process and correct variations. When a significant number of units are detected or measured to be far from the desired target a corrective action has to be carried out to assure that all produced units are, up to a certain point, identical. However, nothing can be done to undo those units detected as defective which will have to be assumed as another cost of the process.*

*Different from classical approaches that consider a system as a black-box whose input can be adjusted to control the output; a forward-looking controller opens and separates that black-box in two sub-systems, the feeding sub-system and the controlled one, as to correct the variations from inside. The idea is simple, instead of detecting and discharging defective units, the controller is meant to correct the deficiencies in a following production step avoiding rejections.*

*By means of implementing simulation algorithms based on Monte Carlo Methods, a forward-looking control has been applied to an idealized micro-assembly process characterized by both a low capability index (Cp<1.33) and distribution mean heavily shifted from the target. The data retrieved from the feeding sub-system was used to vary dynamically the parameters of the controlled sub-system as to keep the process under control and reduce the number of rejections.*

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## 1. Introduction

A mayor concern of serial production is to meet the specifications to ensure that parts are so nearly identical that they can be interchange without any customization. This is important for the assembly new devices as well as the repair the existing ones.

As the micro-production of parts is usually complicated and required sophisticated equipment and high-qualified labor, when a micro-device breaks down the only alternative to costly and probably unpractical repair is to replace it by a new one. The elimination of the need for repairing parts offers the opportunity to explore innovative production methods for those processes whose final products are meant to be dischargeable.

## 2. Tolerance and Capability Indexes

Engineering tolerance can be seen as the range in which the values of a certain dimension are permitted to fluctuate. In the case of assemblies it is necessary to consider the tolerance of each part and the way in which they will be stacked [1].

Considering the functional demands the designer decides upon the tolerances that the assembled product has to meet. In practice today often the assumption is made that all deviations might add in the worst possible way. Then the tolerances of the single parts add up to give the tolerance of the assembly. If the tolerance of the assembly is given, this approach may lead to very tight tolerances of the single parts.

The statistical stacking or RSS (Root Sum Square) method takes into account the most probably the deviations of the single parts cancel each other at least partially, because they are statistically independent. The relation of the tolerance of the assembly and the single part tolerances $t_i$ is:

$$t_{stat} = \sqrt{\sum_{i=1}^{n} t_i^2} \tag{1}$$

Capability indexes are ratios used to compare the standard variation of a dimension of interest with the specification limits LSL (lower) and USL (upper) that define the tolerance band. The potential capability index $C_p$ indicates the theoretical process capability of meeting the specification. It is given by the following equation:

$$C_p = \frac{USL - LSL}{6\sigma} \tag{2}$$

If the mean μ of a given distribution is not on target, it is necessary to take the shift into account and then the actual capability index $C_{pk}$ has to be used instead. It is given by the following equation:

$$C_{pk} = \min\left\{\frac{\mu - LSL}{3\sigma}, \frac{USL - \mu}{3\sigma}\right\} \tag{3}$$

It is important to have a capability index as high as possible to ensure the viability of a given process, technically and economically speaking. It is commonly accepted a process as capable when its potential capability index ($C_p$) is higher then 1.33. A lower $C_p$ would require the process to be reviewed and re-engineered. Nevertheless, in some cases by means of applying the dynamic tolerances ideas the problem of a low $C_p$ can be overcame.

## 3. Dynamic Tolerances

Based on the statistical tolerance stacking, the method of the dynamic tolerances for assemblies indicates how the value of the nominal tolerances can be adjusted as to improve the capability of a certain process. To accomplish this it is necessary to count on enough knowledge about the actual stacked values of the assembly under production. This will be demonstrated with a simple example: Two components with nominal length $L_1$ and $L_2$, respectively are assembled to give an assembly with length $L_{assy}$:

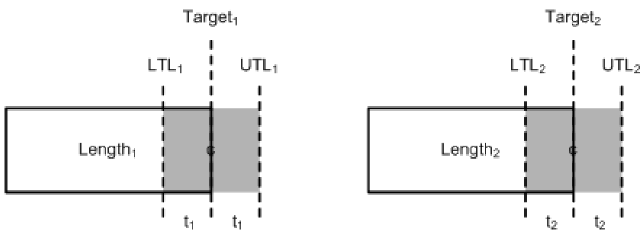


Fig. 1 Specifications of Component 1 and Component 2

The result of putting these two parts together is shown in the next figure. The resultant length is equivalent to the sum of the contributing lengths and the final tolerance is obtained from the equation for statistical tolerance stacking.

$$L_{assy} = L_1 + L_2 \tag{4}$$

$$t_{assy}{}^2 = \sum_{i=1}^{n} t_i^2 \tag{5}$$

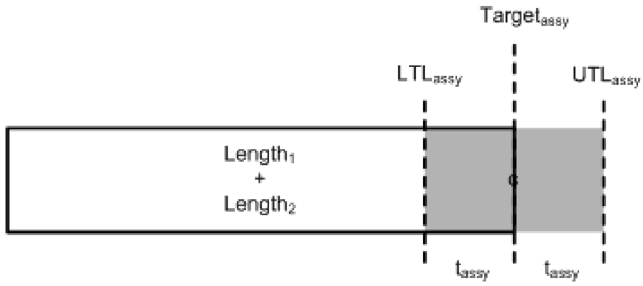


Fig. 2 Cumulative length and tolerance

Obtaining the actual values of the cumulative length is essential to attempt any dynamic adjustment of specifications. If the actual length of the Component 1 is known, as shown in the next figure, then it would be possible to adjust the specification of the Component 2 as to meet the desired length for the assembly.

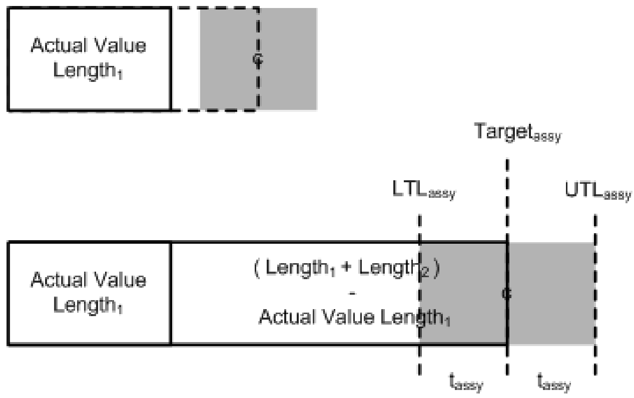


Fig. 3 Adjusted length for the Component 2

From the picture above it can seen that once the actual value has been obtained the data about the tolerance of the Component 1 will not be relevant anymore. Instead, Component 2 can be defined using the full tolerance defined for the assembly.

The consequences of applying dynamic tolerances are not minor. Firstly, it could help to improve the capability of a process without re-engineering it. And secondly, it would make possible to reduce the rejection of parts because it can be applied even if a part is actually out of tolerance.

Not doubt the most remarkable result of dynamic tolerances is the flexibility offered by the adjusted or extended tolerances. In this example, the Component 2 will end up enjoying the full statistical stacked tolerance determined for the assembly. This advantage however has to be paid for by the additional 100% measurement process that is required to determine the length of each of the component 1. Component 2 is then manufactured to fit component 1 individually and that means a step back from industrial production to handicraft. It will be shown in the next section that this can be overcome in many cases by the innovative approach of forward-looking control

## 4. Forward-Looking Control Model (FLCM)

There are some particular characteristics that let FLCM apart from the traditional control approaches. Perhaps the most important and necessary of them is the identification and definition of two sub-systems within the system under control. The sub-systems must be defined in such a way that the system can be split into two parts as to make possible the introduction of an additional measurement between them to retrieve data about the actual values of the system variables.


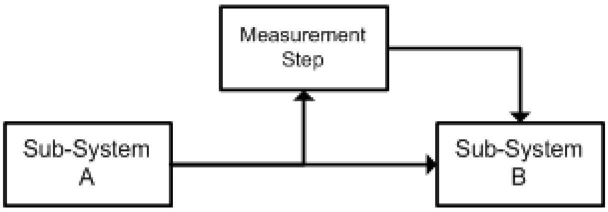


Fig. 4 Sub-systems A & B and additional measurement step

Having the ability to retrieve actual values of the output variables of Sub-system A is an indispensable condition due to the fact it is the only way to apply corrective countermeasures during the inner work of Sub-system B to ensure that the final output is as close as possible to the desired one. The measurement does not have to cover 100% of the output of process A but can be reduced to samples, similar to a classical SPC.

Whereas in the closed-loop controller model the system is considered like a black-box whose inputs can be adjusted as to diminish the difference between the output and the reference; in the FLCM the black-box is replaced by two sub-systems whose output and input are measured and adjusted respectively.

FLCM aims to reduce the variation of a process and to produce a better output by applying correcting countermeasures from the inside of the system. This implies that some of the system's internal parameters will vary in time in order to generate the combined effect that will accommodate the output in such a way that its difference with respect to the reference is minimized.

## 5. Simulation

### 5.1 Initial Assumptions

To apply the concepts presented before the production of a micro-assembly will be simulated. The assembly will consist of three components, namely 1, 2 and 3, whose specifications are identical and which are produced by Process 1, 2 and 3 respectively. For simplicity, all the considerations related to the uncertainty of the measured values resulting from the simulation will be left apart.

Before proceeding with the simulation is necessary to state several assumptions that will be taken for granted in the following sections.

- **Normality**. All of the components' samples obey to normal distributions that are characterized by a certain mean and a standard deviation.
- **No correlation**. Process 1, 2 and 3 belong to different production lines. Thus, there is not reason to consider any correlation among them.
- **Process variation**. Similar to SPC we assume the statistical variations of the processes to be separable into a short term noise (e.g. caused by mechanical vibration) that cannot be controlled at all and a variation on la longer time scale (e.g. caused by temperature variation) that are potentially controllable.

### 5.2 Design Specifications

For the purpose of this exercise, a lot of one thousand micro-assemblies with the following specifications will be simulated:

| | Length [mm] | Tolerance [mm] |
|---|---|---|
| Assembly | 30.00 | 1.00 |

Table 1 Assembly specifications

Once the dimensional specifications for the assembly are defined it is then possible to design its components.

| | Length [mm] | Tolerance [mm] |
|---|---|---|
| Component 1 | 10.00 | 0.58 |
| Component 2 | 10.00 | 0.58 |
| Component 3 | 10.00 | 0.58 |

Table 2 Component specifications

Whereas the length of each component can be estimated with relatively little effort, the corresponding value for their tolerances is not really obvious and demands the application of the equation (1) to be understood.

### 5.3 Process Characterization

Perhaps the most important of the initial assumptions is that one related to the normality of the probability distributions characterizing the production processes of Process 1, 2 and 3. The following table summarized these characteristics and the corresponding indexes $C_p$ and $C_{pk}$ for each process.

| | Mean [mm] | St. Dev. [mm] | $C_p$ | $C_{pk}$ |
|---|---|---|---|---|
| Process 1 | 9.75 | 0.15 | 1.29 | 0.73 |
| Process 2 | 9.85 | 0.20 | 0.97 | 0.72 |
| Process 3 | 9.95 | 0.15 | 1.29 | 1.18 |

Table 3 Process characterization and capability indexes

### 5.4 Rejections

A rejection occurs when a unit or part is measured to be out to tolerance. Consequently only bad units are supposed to be rejected however there will be always a chance to reject a good unit and to accept a bad one. As it can be inferred from previous sections, the number of rejections will increase as the capability indexes decrease. The following table presents the number of rejections resulting from simulating samples of thousand units for each process.

| | Rejections |
|---|---|
| Process 1 | 25 |
| Process 2 | 30 |
| Process 3 | 1 |

Table 4 Rejections per thousand units

## 6. Assembling Components

A remarkable and particularly opportune statistical property states that the sum of two normally distributed random variables, $X_i(\mu_i, \sigma_i)$, will result in another normal distribution whose mean μ and variance $\sigma^2$ are given by the following equations:

$$\mu = \sum_{i=1}^{n} \mu_i \tag{6}$$

$$\sigma^2 = \sum_{i=1}^{n} \sigma_i^2 \tag{7}$$

As it was established in the initial assumptions, Components 1, 2 and 3 are fabricated in different production lines and consequently their samples can be considered as not correlated random variables.

The following table presents the mean and the standard deviation resulting from the linearly combining the probability distribution of Components 1, 2 and 3. The read of these results is rather simple, a sample of thousand final assemblies is expected to be characterized by N(29.55 , 0.30). In this case the resulting distribution will always have a higher standard deviation which means that the complete process o system will have always a joint variation that is higher than those ones of the individual contributors.

| | Mean [mm] | St. Dev. [mm] | $C_p$ | $C_{pk}$ |
|---|---|---|---|---|
| Assembly | 29.55 | 0.30 | 1.13 | 0.62 |

Table 5 Process characteristics and capability indexes of original assembly

## 7. Dynamic Specifications for Component k-th

Due to their nature manufacturing processes are usually under the influence of a wide spectrum of variation sources that in the literature are grouped into five main categories, namely materials, methods, measures, environment and people.

Variations other than the random one are essential and a requisite to start thinking in FLCM because the mere random variation would complicate in extreme the adjustment of parameters that FLCM demands. For this reason output of the feeding Sub-system A has to present a detectable variation.

Sub-system B, in this exercise, is constituted solely by Component 3. So the challenge is finding a proper and feasible way to adjust the input to Sub-system B with the help of the data retrieved from Sub-system A. In this case the parameters to adjust are nothing else but the specification of Component 3. Here it has been generically called Component k-th as to reinforce the fact that any component can be adjusted.

The idea is rather simple; as the data about the cumulative dimensions of Component 1 and 2 are retrieved, the specification for Component 3 will be adjusted. Naturally, it is reasonable to think that modifying the specification for every unit would be neither technically feasible nor realistic and therefore a different approach has to be adopted instead.

## 8. Look-up Window

The look-up window is meant to define the number of units taken from the output of Sub-system A that will be considered at once to estimate the adjustment needed for the input to the Sub-system B. Not all of these units are expected to be measured only a few of them will be considered to obtain a representative value of the units.

For the simplicity of this exercise and to have always a minimum of ten adjustments, in this case the look-up window will be defined to be equal to a tenth of the production lot. As the lot size was arbitrarily defined to be one thousand, the look-up window will be set to be one hundred. Logically, it would not be practical to measure all of the units contained in each look-up window but a representative sample of it instead.

### 8.1 Look-up Window Sample

The most accurate representation of a look-up window would be that one obtained by measuring all of the units contained in it. However, such approach could demand an enormous effort. Therefore in this work, a fifth of the units in the look-up window were measured as to be representative its content. The selection of the units to measure was done randomly.

With the help of a customized algorithm to reduce noise and to interpolate point, the complete content of the look-up window was reconstructed. The algorithm was developed in such a way that all the historical data corresponding to the previous look-up windows was considered to reconstruct the content of current one as to help detecting eventual long time variation in the complete lot.

Figure 5 presents the difference between measuring all units (upper) and a fifth of them (lower).

The specifications for Component 3 will be adjusted according to the average of the last hundred actual values of the assembly of Component 1 and Component 2 as to meet the desired target for the whole assembly. The following are the equations governing these calculations. Let $L_i$ and $T_i$ be the length and tolerance respectively.

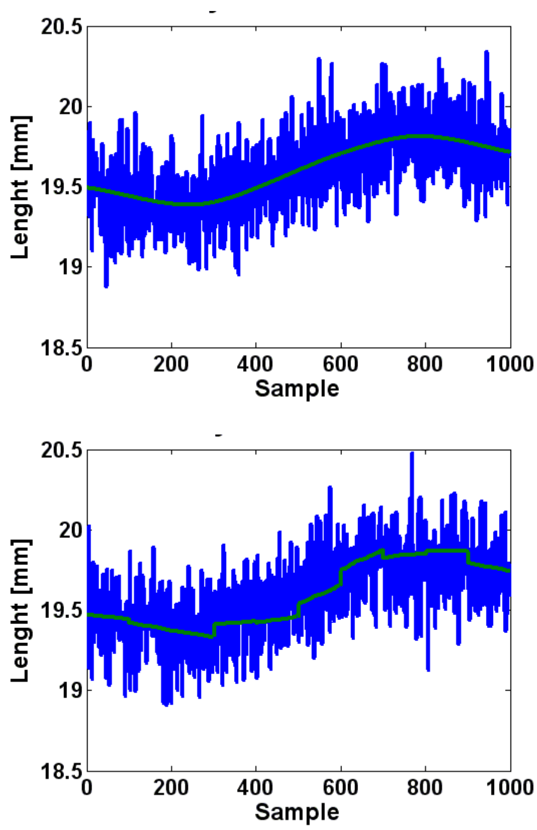


Fig. 5 Comparison of measured units

$$L_{123} = L_1 + L_2 + L_3 \tag{8}$$

$$L_{actual,12} = L_{actual,1} + L_{actual,2} \tag{9}$$

$$L_{adjtd,3} = L_{123} - L_{actual,12} \tag{10}$$

$$T_{3,adj} = \sqrt{t_1^2 + t_2^2 + t_3^2} \tag{11}$$

As the adjusted specification depends on the values obtained from the assembly of Component 1 and Component 2, the sample of Component 3 cannot be considered as an independent variable anymore. Hence, another component has to be considered: the correlation existing between these samples. The valid equations will be now:

$$\mu_{adjtd,assy} = \mu_1 + \mu_2 + \mu_{adjtd,3} \tag{12}$$

$$\sigma_{12} = \sqrt{\sigma_1^2 + \sigma_2^2} \tag{13}$$

$$\sigma_{adjtd,123} = \sqrt{\sigma_{12}^2 + \sigma_{adjtd,3}^2 + 2\rho\sigma_{12}\sigma_{adjtd,3}} \tag{14}$$

Where $\rho$ is the correlation between the samples of Component 1 and Component 2.

It turns out of mayor interest to know how exactly the adjustments made to the specifications of Component 3 evolve through the time. This is shown in the following figure.

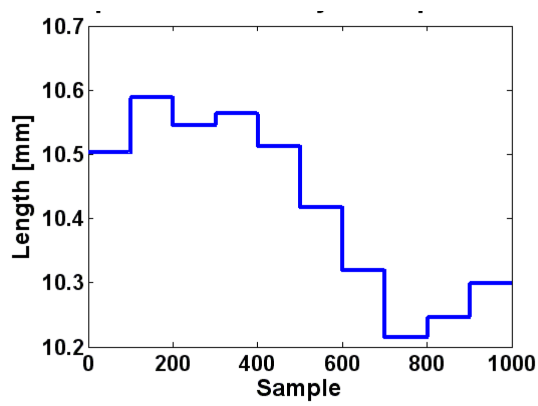


Fig. 6 Component 3 - Specification adjustments

Although exaggeratedly zoomed, the curve described clearly the expected pattern. The specification of Component 3 has been adjusted as to balance the variation existing in the assembly of Component 1 and Component 2.

## 11. Comparison of Histograms

To realize in a better way the impact of applying FLCM it is useful to compare both the original and the adjusted histogram. This is shown in the following figure, where the curve on the right represents the distribution after applying FLCM.

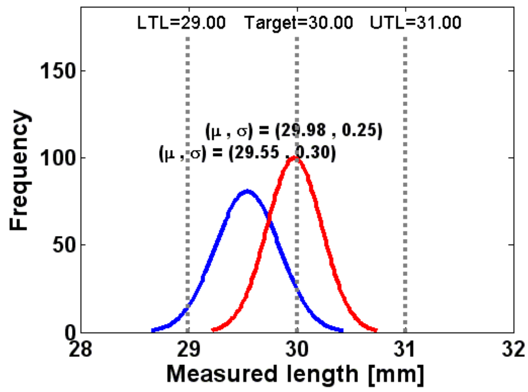


Fig. 7 Assembly – Histogram – Original (left) and controlled (right)

In fact, as it was expected there is a mean shift which at the first sight seem to bring some good benefits as the mean now lies really close to the target. There is also a reduction in the standard deviation.

### 11.1 Process Capability and Rejections

It is interesting to know what happen to the capability indexes and the number of rejections as a result of all the interactions carried out. The results are presented in the following table.

| Assembly | $C_p$ | $C_{pk}$ | Rejections |
|---|---|---|---|
| Original | 1.16 | 0.63 | 32 |
| Controlled | 1.36 | 1.28 | 0 |

Table 12 Capability indexes and rejections comparison

As the standard deviation varied after the application of FLCM, the indexes are affected as well. Most significant is the case of the actual capability index is different as it takes the shift of the mean into account.

Even though the process still exhibits not particularly high capability indexes, the improvement in the process' results is undeniable. This can be seen directly in the histogram where the region out of the tolerance zone has been considerably reduced or in the number of rejected assemblies per thousand opportunities.

## 12. Conclusions

Through out this work the principles of FLCM have been extensively explained and with the help of a simple example a number of important results have been revealed. The connection existing between FLCM and Dynamic Tolerance concepts has been also deeply exploited to make the reader understand the way in which they interact.

The following are the main conclusions of this work. They have been categorized and prioritized according to their relative relevance.

### 12.1 FLCM

- Different from classical approaches that are design to detect and to discharge defective units; FLCM is expected to counteract their deficiencies by adjusting the process parameters and avoid discharging them.
- Classical approaches considered the system as a black-box whose input can be adjusted as to control the output. Instead, FLCM needs the black-box to be open as to control the system from inside.
- A significant improvement in the mean of the system's distribution is likely expectable. Given that FLCM counteracts and correct the system from inside, the resultant mean will be closer to the target than the mean of the non-controlled system.
- The action of moving the mean of the distribution to reduce the shift from the target will necessarily result in a reduction of the number of rejection as a greater part of the area under the distribution curve will lye within the tolerance zone.

### 12.2 Dynamic Tolerances

- Larger tolerances have two main advantages. Firstly, from the corresponding equation it can be seen that the capability indexes will be higher. And secondly, it can be expected a reduced number of rejections as a direct consequence a have extended tolerance limits.
- Having dynamic tolerances will certainly lead to produce unique parts that are expected to fit in under specific conditions.

### 12.3 Look-up Window

- The size of the look-up window certainly will have an impact in the final result. As the smaller the size the better the fitting to the variation curve. However it is necessary to evaluate whether the effort will pay-off or not.
- A good approach would be having an intelligent algorithm to detect any pattern in the sample and to adapt dynamically the windows size.

### 12.4 Look-up Window Sample

- It is not doubt crucial to know how many of the units have to be measured to obtain a representative value at a reasonable effort.

Finally, the general conclusion of this work is that the application of FLCM to a system that fulfils the requisites explained in these pages can lead to a reduction of both the standard deviation and the mean and consequently to a reduction in the number of the rejected units. By means of a simple example and computer simulations the ideas of correcting the system from inside and of balancing the deficiencies of defective components as to complete non-defective assemblies have been satisfactorily shown.

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