# Development of a multi-degrees-of-freedom laser encoder using $\pm 1$ order and $\pm 2$ order diffraction rays 

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#### Abstract

We developed a multi-degrees-of-freedom laser encoder to simultaneously measure the position, straightness, pitch, roll, and yaw errors of a linear stage. This study integrated the interferodiffractive technique with the three-dimensional diffracted ray-tracing method to develop a novel laser encoder with multi-degrees-of-freedom. T he linear displacement was measured from the phase encoded of +1 and -1 order diffraction rays reflected from a moved diffraction grating on a linear stage using interferodiffractive technique. The diffracted direction of the +2 and -2 order diffraction rays changed by the motion errors of the moved grating were analyzed to inver sely calculate the other motion errors based on the detection of light spots on two quadrant photodiode detectors. The period of the used grating is $4 \mu m$ and the experimental results show that the measurement accuracy was better than $\pm 0.6 \mu \mathrm{~m} / \pm 50 \mu \mathrm{~m}$ for straightness, $\pm 1$ arc sec $/ \pm 20$ arc sec for angular error components, and $\pm 0.9 \mu \mathrm{~m} /$ $\pm 500 \mu \mathrm{~m}$ for linear displacement.


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## 1. Introduction

The optical linear encoders with one dimensional position measurement can significantly improve the displacement precision in the CNC machine tools [1-4]. Optical encoders based on different principles, such as Moiré [1, 2, 5], Lau effect [6], and interferodiffractive $[3,7,8,9,10]$ have been developed. The researches for one dimensional linear encoder on the metrological errors such as the lack of quadrature, unequal gain in the detector channels, zero offset, cyclic error of a circular polarization interferometer with a quadrature detector system, temperature variations, vibrations, errors in the replication of the gratings, have been published in the past years [11,12,13,14]. In recent years, the two-dimensional optical linear encoders were developed rapidly and can provide two-dimensional positioning and high resolution and accuracy $[1,2,4,10]$. However, there are six geometric errors of one moving axis in CNC machine tools. For a three-axis CNC machine tools, the effects of a component's error on the position error of the toolpoint or the workpiece which are induced from twenty one geometric errors. In order to find the solution for real time measurement or monitor of the multi-degrees-of-freedom motion errors, our previous study [15] first proposed a five-degrees-offreedom laser encoder using the interferodiffractive technology and diffracted ray tracing method. In that laser encoder system, the HeNe laser was used as a laser source and the $+/-1$ order diffracted rays were separated into two parts; one is for interference and another is for diffracted ray tracing. This optical design make the system is large and difficult to be miniaturized. In this study, a modified and miniaturized 5-DOF laser encoder is proposed and a laser diode was used as a laser source and two high order diffracted rays were used
for measuring three angular error, pitch, yaw and roll. In the first par t of this paper, the optical design and theory analysis were clearly described. In the second part, each measured elements was successfully verified using the HP laser interferometer calibration.

## 2. The principle of the system

### 2.1 Overall System Configuration

The overall system configuration includes a five-degrees-of-freedom optical encoder head and a diffraction grating scale was shown in Fig.1. T The five-degrees-of-freedom optical encoder head is composes of a laser diode of the light source and two reflection mirrors, a $\lambda / 4$ wave plate, a beam splitter, two polarizers, two photodiodes and two quadrant detectors. The grating scale is a one-dimensional reflection diffraction grating which is the medium use to deliver the signal of the stage motion. Consequently, this part will install on a moving stage. When the stage is moving, the associated $\pm 1$ order and $\pm 2$ order diffracted rays generated from the grating scale can be received by the optical head. That is use the phase difference of the $\pm 1$ order diffraction rays to measure linear displacement by interference principle. The space information of the $\pm 2$ order diffraction rays detected by two quadrant detectors will use to measure three angular motions (pitch, yaw and roll) and a linear error base on the ray tracing method. That will be able to constitute a multi-degrees-of-freedom laser encoder. The detail drawing of the optical design can be seen in Fig.1.
The laser diode emits a laser beam through the mirror 1 and then the reflected beam is projected onto the grating scale than produce $\pm 1$ order and $\pm 2$ order diffraction ray (the diffraction angle is about 9.35 degrees and 19 degrees). The $\pm 1$ order diffraction ray is designed to measure the linear displacement and the $\pm 2$ order diffraction rays is designed to measure one straightness error and three angular motion
errors. The $\pm 1$ order diffraction rays reflected by the mirror 2 , the mirror 3 and only the +1 order diffraction ray will pass through a $\lambda / 4$ wave plate to generate phase retardation. Afterward they will generate a group of interference signals and make two signals receive d by two photodiode phase difference at 90 degrees using two polarizers. The signals of two photodiodes are electronically interp olated to achieve high linear resolution. The $\pm 2$ order diffraction rays are directly received by the quadrant photodiode detector 1 and quadrant photodiode detector 2. Use for three-dimensional laser tracking technology and inverse technology, the straightness error and the pitch, yaw and roll error can be calculated.


Fig. 1 Optical design configuration of the 5-DOF laser encoder.

### 2.2 Measuring Principle of Linear Displacement

The electric fields of $\pm 1$ order diffraction rays can be represented as follows:
$E_{+1}=E_{o}[\cos (\omega t) \hat{i}+\cos (\omega t) \hat{j}]$
$E_{-1}=E_{o}[\cos (\omega t) \hat{i}+\cos (\omega t) \hat{j}]$
Where Eo is the amplitude, $\omega$ is the optical frequency, $i$ and $j$ are two unit vectors of orthogonal polarization direction, the $\lambda / 4$ wave plate provides 90 degrees phase retardation, thus the electric field of +1 order diffraction ray can be expressed as
$E_{+1}=E_{o}\left[\cos (\omega t) \hat{i}+\cos \left(\omega t+90^{\circ}\right) \hat{j}\right]$
Considering the Doppler effect of the moving scale, the $\pm 1$ order diffraction rays have the phase shift of $\Delta \phi$ and their electric fields ca be expressed as

$$
\begin{align*}
& \mathrm{E}_{-1}=\mathrm{E}_{\mathrm{o}}[\cos (\omega \mathrm{t}-\Delta \phi) \hat{\mathrm{i}}+\cos (\omega \mathrm{t}-\Delta \phi) \hat{\mathrm{j}}]  \tag{3}\\
& \mathrm{E}_{+1}=\mathrm{E}_{\mathrm{o}}\left[\cos (\omega \mathrm{t}+\Delta \phi) \hat{\mathrm{i}}+\cos \left(\omega \mathrm{t}+\Delta \phi+90^{\circ}\right) \hat{\mathrm{j}}\right] \tag{4}
\end{align*}
$$

After $\pm 1$ order diffraction rays pass through the beam Splitter and the polarizer 1 and polarizer 2 respectively, the electric fields of the interference signals receive by the photodiode 1 and photodiode 2 are express as:

$$
\begin{align*}
& E_{\text {Photodiodd }}=E_{o}[\cos (\omega t-\Delta \phi)+\cos (\omega t+\Delta \phi)] \hat{i}  \tag{5}\\
& E_{\text {Photodiode }}=E_{o}\left[\cos (\omega t-\Delta \phi)+\cos \left(\omega t+\Delta \phi+90^{\circ}\right)\right] \hat{j} \tag{6}
\end{align*}
$$

where the optical intensity can be represented as follows:
$I_{\text {Photodiodd }}=E_{o}{ }^{2}[1+\cos (2 \Delta \phi)]$
$I_{\text {Photodiode }}=E_{o}^{2}\left[1+\cos \left(2 \Delta \phi+90^{\circ}\right)\right]$
The relation of the displacement $\Delta x$ and phase $\Delta \phi$ is
$\Delta x=\frac{\Delta \phi \times d}{2 \pi}$

### 2.3 Kinematic analysis of the diffracted ray

The orientation of the grating reflects the directional variation of th e $\pm 2$ order diffraction rays. Two quadrant photodiode detectors are used to receive the incident position of the $\pm 2$ order diffraction ray s. Thus, four position signals from two quadrant detectors can be i nversely calculated the error of straightness, pitch, yaw and roll usin g kinematic analysis. That symbols can be expressed $\Delta \mathrm{z}, \theta \mathrm{x}, \theta \mathrm{y}$ and $\theta z$ shown in Fig.2.

In order to analyze the spatial relationship between the orientatio n variance of the grating and their corresponding spot positional data of the quadrant photodiode detectors, both forward and inverse kinematic analyses are required. The forward kinematic analysis provides the relationship between the grating and their corresponding quadrant detectors spot positional data. The inverse kinematic analysis calculates the motion error from the variance of quadrant detector spot positional data using the multidimensional Newton-Raphson iteration algorithm. Fig .3 shows the expression of a homogeneous transformation matrix between coordinate frames $\{\mathrm{G}\}$ and $\{\mathrm{R}\},{ }^{\mathrm{R}} \mathrm{T}_{\mathrm{G}}$, can be expressed as follows
${ }^{R} T_{G}=\left[\begin{array}{cccc}\mathrm{c} \alpha \mathrm{c} \beta & \mathrm{c} \alpha \mathrm{s} \beta \mathrm{s} \gamma-\mathrm{s} \alpha \mathrm{c} \gamma & \mathrm{c} \alpha \mathrm{s} \beta \mathrm{c} \gamma+\mathrm{s} \alpha \mathrm{s} \gamma & \mathrm{p}_{\mathrm{x}} \\ \mathrm{s} \alpha \mathrm{c} \beta & \mathrm{s} \alpha \mathrm{s} \beta \mathrm{s} \gamma+\mathrm{c} \alpha \mathrm{c} \gamma & \mathrm{s} \alpha \mathrm{s} \beta \mathrm{c} \gamma-\mathrm{c} \alpha \mathrm{s} \gamma & \mathrm{p}_{\mathrm{y}} \\ -\mathrm{s} \beta & \mathrm{c} \beta \mathrm{s} \gamma & \mathrm{c} \beta \mathrm{c} \gamma & \mathrm{p}_{\mathrm{z}} \\ 0 & 0 & 0 & 1\end{array}\right]$

$$
=\left[\begin{array}{cc}
{ }^{R} R_{G} & { }^{R} \vec{p}_{G} \\
0 & 1
\end{array}\right]
$$

where $\cos$ and $\sin$ are abbreviated as c and s for simplicity.[ $\mathrm{px}, \mathrm{py}$, pz ] can be expressed as relative to translation vector of $\mathrm{x}, \mathrm{y}$ and z axis and $[\alpha, \beta, \gamma]$ are rotational angle. ${ }^{\mathrm{R}} \mathrm{T}_{\mathrm{G}}$ can be decomposed into a $3 \times 3$ rotational matrix ${ }^{\mathrm{R}} \mathrm{R}_{\mathrm{G}}$ and a $3 \times 1$ translation vector ${ }^{\mathrm{R}} \overrightarrow{\mathrm{p}}_{\mathrm{G}}$ as presented in Eq.(10). The unit vector ${ }^{\mathrm{Rl}}{ }_{\mathrm{S}}$ of the incident ray to the grating can be considered to propagate from the origin of $\{\mathrm{S}\}$ to that of $\{G\}$. Here $\{R\}$ is the reference coordinate system.

The incident ray coordinate system $\{S\}$ can be transformed to the grating coordinate system $\{G\}$ by means of Eq. (10), and th e unit vector of the $\mathrm{m}_{\text {th }}$-order diffracted ray ${ }^{\mathrm{G}} 1_{\text {mgo }}$ can all be expressed as follows.
${ }^{G} l_{S}={ }^{G} R_{R}{ }^{R} l_{S}=\left[\begin{array}{llll}l_{s x} & l_{s y} & l_{s z} & 1\end{array}\right]^{T}$
${ }^{G} l_{m g o}=\left[\begin{array}{c}l_{m g x} \\ l_{m g y} \\ l_{m g z} \\ 1\end{array}\right]=\left[\begin{array}{c}l_{s x}+m \frac{\lambda}{d} \\ l_{s y} \\ \sqrt{1-l_{m g x}^{2}-l_{m g y}^{2}} \\ 1\end{array}\right](m= \pm 2)$
where a rotation matrix ${ }^{G} R_{R}$ describes $\{\mathrm{R}\}$ relative to $\{\mathrm{G}\}$.
After that the relationship between $\{\mathrm{G}\}$ and spatial positions of t he quadrant detectors can be expressed as ${ }^{\mathrm{Dm}} \mathrm{P}_{\mathrm{go}}$.
${ }^{D m} P_{g o}={ }^{D m} T_{R}{ }^{R} P_{g o}=\left[\begin{array}{llll}x_{m g o} & y_{m g o} & z_{m g o} & 1\end{array}\right]^{T}(m= \pm 2)$
where ${ }^{D m} T_{R}$ can be expressed as the translation matrix of $\{\mathrm{R}\}$ relative to of the coordinate system $\{\mathrm{Dm}\}$ of the quadrant detector and ${ }^{R} P_{g o}$ is coordinate of $\{G\}$ relative to $\{R\}$.

The diffraction rays ${ }^{\mathrm{Dm}} \mathrm{l}_{\mathrm{G}}$ expressed as $\{\mathrm{G}\}$ relative to $\{\mathrm{Dm}\}$ can be written as
${ }^{D m} l_{m g o}={ }^{D m} R_{R}{ }^{R} R_{G}{ }^{G} l_{m g o}=\left[\begin{array}{llll}l_{m x} & l_{m y^{\prime}} & l_{m z^{\prime}} & 1\end{array}\right]^{T}(m= \pm 2)$
Finally, the incident positions coordinate on the $\mathrm{m}_{\mathrm{th}}$-order quadrant detector of can be expressed as
$\mathrm{x}_{\mathrm{ml}}=\mathrm{x}_{\mathrm{mgo}}-\frac{1_{\mathrm{mx}^{\prime}}}{1_{\mathrm{mz}^{\prime}}} \mathrm{z}_{\mathrm{mgo}}$
$\mathrm{y}_{\mathrm{ml}}=\mathrm{y}_{\mathrm{mgo}}-\frac{\mathrm{l}_{\mathrm{my}}}{\mathrm{l}_{\mathrm{mz}}} \mathrm{z}_{\mathrm{mgo}}$
$\mathrm{m}= \pm 2$
The inverse kinematic of Eqns. (15) can be calculated using Newton-Raphson iteration algorithm


(c) Linear motion along z-axis( $\Delta \mathrm{z}$ )

(e) Angular motion about $y-\operatorname{axis}(\theta y)$

(d) Angular motion about x -axis $(\theta \mathrm{x})$

(f) Angular motion about z - $\operatorname{axis}(\theta \mathrm{z})$

Fig. 2 Detection of quadrant detectors according to orientation variaiton of the diffraction grating.


Fig. 3 Coordinate definition for the $\mathrm{m}_{\mathrm{th}}$-order diffracted ray.

## 3. Experimental results and discussion

In the system, we used Heidenhain LIP 400 series linear grating scale with pitch of grating is 4 micrometer, and $\pm 1$ order and $\pm 2$ order diffraction angular are $9.35^{\circ}$ and $19.00^{\circ}$. The optical lens in DVD pickup head, additional two polarizers and two photodiodes are us ed in the interference system. Use CEL-5/60I (Elmo 1.t.d) to perfor m the electronic interpolation, it can promote resolution higher to $1 / 128$ micrometer. The spot positions of the diffracted rays are me asured by the quadrant photodiode detector (Hamamatsu ,S4349 ). To eliminate the effect of environmental vibration, the experimental systems were all set up on an ant vibration optical table in a temperature controlled laboratory.

Fig. 4 shows the interference signals of the photodiode 1 and photodiode 2, that can know two signals orthogonal situation (whether for the perfect circle), amplitude and DC term by Lissajous circle.


Fig. 4 The interference signal measured by the oscillograph

The verification of the linear displacement, straightness error and the angular errors of the roll, yaw and pitch were performed by using HP5529A laser interferometer with $0.01 \mu \mathrm{~m}$ of linear resolution and 0.01 arcsec of the angular resolution.

The calibration results are shown in Figs.5-9. The experiments we re repeated three times to ensure the measurement repeatibility.

The calibration range for linear displacement $(\Delta x)$ is less than $\pm 500$ micrometer. The measurement accuracy for linear displacement is $\pm 0.9$ micrometer.


Fig. 5 Linear displacement measurement ( $\Delta \mathrm{x}$ ): comparison between the HP 5529A laser interferometer and the system.

The calibration range for straightness ( $\Delta \mathrm{z}$ ) is less than $\pm 50$ micrometer. The measurement accuracy for straightness is $\pm 0.6$ micrometer.


Fig. 6Linear motion along z -axis $(\Delta \mathrm{z})$ : comparison between the HP 5529A laser interferometer and the system.

The calibration range for angular measurement $(\theta \mathrm{x})$ is less than $\pm 50$ arc sec. The measurement accuracy for pitch is $\pm 1$ arc sec.


Fig. 7 Angular motion about x -axis $(\theta \mathrm{x})$ : comparison between t he HP 5529A laser interferometer and the system.

The calibration range for angular measurement ( $\theta \mathrm{y}$ ) is less than $\pm 20$ arc sec. The measurement accuracy for yaw is $\pm 1$ arc sec.


Fig. 8 Angular motion about $y$-axis ( $\theta \mathrm{y}$ ): comparison between the HP 5529A laser interferometer and the system.

The calibration range for angular measurement $(\theta \mathrm{z})$ is less than $\pm 80 \mathrm{arc} \sec$. The measurement accuracy for roll is $\pm 1 \mathrm{arc} \mathrm{sec}$.


Fig. 9 Angular motion about z -axis $(\theta \mathrm{z})$ : comparison between the HP 5529A laser interferometer and the system.

## 4. Conclusions

The paper proposed a high resolution multi-degree-of-freedom 1 aser linear encoder with linear displacement ( $\Delta \mathrm{x}$ ), straightness error ( $\Delta \mathrm{z}$ ) and roll, yaw and pitch e rrors. It is already successfully developed and modularity so that it can be realized for using in the CNC machine tools for on machine measurement and monitor. In the future, this system will be us ed for measuring the multi-degrees of freedom thermal deformati on of CNC machine tools.

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