Uncertainty Evaluation in Geometric Length Measurement by CMM based on Monte Carlo Method

Li Hongli [1], **Chen Xiaohuai** [1,#], **Wang Hongtao** and **Cao Xuemei** [2]

1 University lecturer, Department of Instrumental Science and Optoelectronic Engineering, Hefei University of Technology, Hefei, Anhui, 230009, China
2 Department of Instrumental Science and Optoelectronic Engineering , Hefei University of Technology , Hefei, Anhui, 230009, China
Corresponding Author / E-mail: lihongli_hgd@163.com , TEL: +0086-0551-2901631-608, FAX: +0086-0551-2901508

KEYWORDS : Coordinate Measuring Machine (CMM), Geometric Length Measurement, Monte Carlo Simulation, Uncertainty Evaluation, Mathematic Model

Abstract: At present, coordinate measuring machine (CMM) has extensive use in the modern manufacturing industry, it is very convenient to get the estimated value of a measured quantity by CMM, but is difficult to directly give out the corresponding uncertainty. To ensure the use reliability of the measurement result, a complete statement of it must include information about the uncertainty of measurement characterizing its measuring quality. For CMM, there will be many complicated factors influencing its measurement accuracy in the measuring process, so highly attention has always been pay to uncertainty evaluation of the measurement result of CMM. This paper presents a complete uncertainty evaluation process of geometric length measurement by CMM. To begin with, the major sources of uncertainty, which will influence measurement result, are found out after analyzing, then, the general mathematic model of geometric length measurement is established. Furthermore, Monte Carlo method (MCM) is used, and the uncertainty of the measured quantity is obtained. Finally, the results of uncertainty evaluation obtained from MCM method and from GUM method are compared, the comparison result indicates that the mathematic model is feasible, and using MCM method to evaluate uncertainty is easy and efficient, having practical value. Keywords: Coordinate Measuring Machine (CMM), Geometric Length Measurement, Monte Carlo Simulation, Uncertainty Evaluation, Mathematic Model

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NOMENCLATURE

MCM = Monte Carlo method
GUM = Guide to the Expression of Uncertainty in Measurement

1. Introduction

Coordinate Measuring Machine (CMM) is a high efficient and universal measuring device, mainly used for measuring product size, shape, position and other geometric parameters, it is a Indispensable measuring equipment on inspection, quality control and manufacturing engineering in the modern industry, and has been widely applied in machinery industry, electronic industry, aerospace, defense and military fields. However, to the existing CMM in the application, it is usual that only the estimated value of the measured quantity can be given and its corresponding measurement uncertainty can not. Measurement uncertainty is a parameter Included in the measurement results[1], and is an important indicator characterizing the quality of measurement results. A result without uncertainty is incomplete, senseless, and has no practical value. Therefore, in order to get complete and practical valued measurement result, and enhance the value of CMM, this paper presents the ideal of uncertainty evaluation about geometry measurements when CMM is used for task-oriented geometry measurements.

"Guide to the Expression of Uncertainty in Measurement" (referred to as the GUM) provides the method of uncertainty evaluation, which is based on uncertainty propagation law and mainly apply to linear model[1]. But in the high-precision measurement, complex models are more common, in which it is difficult to find out the linear relationship between input and output, and to show their function with a clear mathematical formula, or is there a wide range of input distribution, etc. When evaluating measurement uncertainty of such complex models with GUM method, there will be many flaws because it is not only difficult to determine the sensitivity and correlation between input quantities, but also difficult to determine the effective degrees of freedom[2]. For this situation, "Supplement 1 to the 'Guide to the expression of uncertainty in measurement' — Propagation of distributions using a Monte Carlo method" provides the guidelines about how to use Monte Carlo method to evaluate uncertainty through propagation of distributions[3].

Monte Carlo method (shortened as MCM) is a computing method based on random numbers. The idea of this method is to build the relation model between input and output quantities, and produce approximate large samples of measured quantity by computer simulation, then get the best estimate value of the measured quantity through statistical analyses and express its accuracy with the standard deviation of the large samples. In the actual measurement process, there will be many random factors influencing measurement results, and the influence of the factors are also random, therefore, it is feasible and more reliable that MCM uses random numbers to simulate actual measurement result, especially reducing the influence of human factor to the minimum. Moreover, MCM is more convenient to use because it has no limit to the type of models and correlation of inputs, and needn't to calculate the partial derivatives and the degree of freedom. In addition, Monte Carlo simulation is easy to achieve through computer software programming, especially for complex systems, it has more practical value.

This paper presents an uncertainty evaluating process of geometric length measurement of CMM. Firstly, the major factors influencing uncertainty of measurement results in whole measurement process are comprehensively analyzed, and task-oriented uncertainty model of CMM is established. Then measurement uncertainty of measured quantity is gotten through Monte Carlo simulation. Finally, the evaluating results obtained from MCM and respectively from traditional GUM method are compared.

2. Measuring model of geometric length

Geometric length measured by CMM is usually obtained using indirect measurement method, i.e. to find the distance between the centers of two corresponding end surfaces in the direction of measured length. In this paper, considering measurement strategies, firstly, n points P_i ($i=1,\ldots,N.$ requesting N≥3) are gotten from one end surface in the measurement direction, and fitted into the corresponding fitting plane V through least squares fitting, then from the center of the other end surface is point P_k (x_k, y_k, z_k) taken, and the distance L from P_k to the fitting plane V is obtained after calculating.

The regression equation of least squares fitting plane V is given by

$$\hat{z} = ax + by + c$$

Corresponding to the N-fitting points P_i ($i=1,\ldots,N.$), the normal equations is given by

$$\begin{pmatrix} N & \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} y_i \\ \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 & \sum_{i=1}^{N} x_i y_i \\ \sum_{i=1}^{N} y_i & \sum_{i=1}^{N} y_i x_i & \sum_{i=1}^{N} y_i^2 \end{pmatrix} \begin{pmatrix} c \\ a \\ b \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{N} z_i \\ \sum_{i=1}^{N} x_i z_i \\ \sum_{i=1}^{N} y_i z_i \end{pmatrix} \quad (1)$$

Where a, b and c can be obtained by solving Eq. (1), the distance from P_k to the fitting plane V can be calculated by

$$L = \frac{|z_k - ax_k - by_k - c|}{\sqrt{a^2 + b^2 + 1}} \quad (2)$$

In order to reduce the influence on measurement results made by repeatability of measurement error, not only N but also m, the numbers of repeated measurement for P_k and P_i, is taken as large as possible in the measurement process, moreover, when calculating the fitting plane V and the distance L, the corresponding mean coordinates and arithmetic standard deviations are taken into.

Because there are many factors influencing measurement results in the actual measurement process, the L obtained above is not the actual best estimate value of measured length, to get it, the various sources of uncertainty influencing measurement results are firstly analyzed as follows

(1) Measurement uncertainty caused by Indication error of CMM. It also reflects the influence which 21 machine errors of CMM and probe error took on the measurement results, it is a major source of uncertainty.

(2) Measurement uncertainty caused by the roughness of measurement surfaces. The influence varies with the roughness of measurand, so whether the factor is a major source of uncertainty or not could be determined by actual measurand.

(3) Measurement uncertainty caused by measurement strategy. Measurement strategy has a direct impact on the measurement results, so it is considered as a major source of uncertainty.

(4) Measurement uncertainty caused by repeatability error of measurement. The influence of it can not be neglected, also as a major source of uncertainty.

(5) Measurement uncertainty caused by environmental conditions of measurement. Since the actual measurement environment (such as temperature) is different from the standard measurement environment (such as temperature) that CMM test required, it will affect measurement results, it also should be considered as a major source of uncertainty;

(6) Measurement uncertainty caused by the deviation of measuring point P_k to the center of measuring surface. This factor also has a direct impact on the measurement results, so it is considered as a major source of uncertainty.

The above analyses can be summed up the general mathematical model about geometry length measurement of CMM:

$$l = [L \cdot (1 + \alpha_w \cdot \delta\theta) + \delta l_r] \cdot (1 - \alpha_s \cdot \delta\theta) + \delta l_{res} + \delta l_v \quad (3)$$

Where L is as the estimated value of measured geometric length got by the measurement method, α_w is as the coefficient of thermal expansion of the measurand, α_s is as the coefficient of thermal expansion of grating of CMM, $\delta\theta$ is as the D-value between ambient temperature of actual measurement and the standard ambient temperature, δl_r is as the influence that roughness of measured surface take to measurement results, δl_{res} is as the influence that Indication error of CMM take to measurement results, δl_v is as the influence that the actual measurement point P_k deviating from the center of measuring surface take to measurement results.

3. Uncertainty of Geometric length measurement

3.1 Measurement Uncertainty Model

The mathematical model of geometric length Eq. (3) shows

$$l = f(L, \delta\theta, \delta l_r, \delta l_{res}, \delta l_v)$$

Its input quantities (L, $\delta\theta$, δl_r, δl_{res} and δl_v) are uncorrelate, according to the uncertainty propagation law of GUM, the transfer coefficients are obtained

$$\frac{\partial f}{\partial L} = (1 + \alpha_w \cdot \delta\theta) \cdot (1 - \alpha_s \cdot \delta\theta)$$

$$\frac{\partial f}{\partial \delta\theta} = L \cdot [\alpha_w (1 - \alpha_s \cdot \delta\theta) - \alpha_s (1 + \alpha_w \cdot \delta\theta)] - \alpha_s \cdot \delta l_r$$

$$\frac{\partial f}{\partial \delta l_r} = 1 - \alpha_s \cdot \delta\theta$$

$$\frac{\partial f}{\partial \delta l_{res}} = \frac{\partial f}{\partial \delta l_v} = 1 \circ$$

The combined standard uncertainty of geometric length measurement L is

$$u_c(l) = \sqrt{\left(\frac{\partial f}{\partial L}\right)^2 \cdot u^2(L) + \left(\frac{\partial f}{\partial \delta\theta}\right)^2 \cdot u^2(\delta\theta) + \left(\frac{\partial f}{\partial \delta l_r}\right)^2 \cdot u^2(\delta l_r) + u^2(\delta l_{res}) + u^2(\delta l_v)} \quad (4)$$

After similarly analyzing according to Eq. (2), $u(L)$ is obtained. Eq. (2) shows

$$L = g(x_k, y_k, z_k, a, b, c)$$

Its input quantities (x_k, y_k and z_k) are uncorrelate, a, b and c are weaker correlate, so the correlations between them are also ignored. According to the uncertainty propagation law of GUM, the transfer coefficients are obtained

$$\frac{\partial g}{\partial x_k} = \frac{-a}{\sqrt{a^2 + b^2 + 1}}; \quad \frac{\partial g}{\partial y_k} = \frac{-b}{\sqrt{a^2 + b^2 + 1}}; \quad \frac{\partial g}{\partial z_k} = \frac{1}{\sqrt{a^2 + b^2 + 1}}$$

$\frac{\partial g}{\partial a}=\frac{-x_k\cdot(b^2+1)}{(a^2+b^2+1)\sqrt{a^2+b^2+1}}$; $\frac{\partial g}{\partial b}=\frac{-y_k\cdot(a^2+1)}{(a^2+b^2+1)\sqrt{a^2+b^2+1}}$; $\frac{\partial g}{\partial c}=\frac{-1}{\sqrt{a^2+b^2+1}}$ 。

Therefore

$$u(L)=\sqrt{\begin{array}{l}\left(\frac{\partial g}{\partial x_k}\right)^2\cdot u^2(x_k)+\left(\frac{\partial g}{\partial y_k}\right)^2\cdot u^2(y_k)+\left(\frac{\partial g}{\partial z_k}\right)^2\cdot u^2(z_k)\\+\left(\frac{\partial g}{\partial a}\right)^2\cdot u^2(a)+\left(\frac{\partial g}{\partial b}\right)^2\cdot u^2(b)+\left(\frac{\partial g}{\partial c}\right)^2\cdot u^2(c)\end{array}} \quad (5)$$

By taking Eq. (5) into Eq. (4) can the model of the combined standard uncertainty of geometric length measurement l be obtained.

3.2 Monte Carlo method for uncertainty evaluation

Can be seen from 3.1, the mathematical model and the combined standard uncertainty models of its geometric length are more complex, in this case, it is more suitable to apply MCM to getting the measurement uncertainty of the geometric length according to its mathematic model[4].

Specific steps are as follows:

(1) To analyze the sources of uncertainty in the measurement process of Geometric length such as $\delta\theta, \delta l_r, \delta l_{res}$ and δl_v, and assume their distribution patterns and intervals;

(2) To determine the expected values and standard deviations of the inputs x_k, y_k, z_k, x_i, y_i and z_i;

(3) To simulate measured value of L according to the random array generated by the expected values and standard deviations of the inputs, taking the sample size as M. M is more large and the result of simulation is more accurate, generally taken as $10^5 \sim 10^6$ to meet the requirement[5].

(4) To similarly simulate the samples of $\delta\theta, \delta l_r, \delta l_{res}$ and δl_v according to their distribution patterns and intervals, taking the sample size as M;

(5) To substitute the M samples random sequences of L, $\delta\theta$, δl_r, δl_{res} and δl_v obtained from the above into the mathematic model of geometric length calculation, and to get M samples of it;

(6) To take the mean of the M samples as the measured geometric length l [6];

(7) To take the standard deviation of the M samples as the standard uncertainty $u(l)$ of the measured geometric length l;

(8) To sequence the the M samples of in ascending order, if given the coverage probability P, the coverage interval of the measurement result can be estimated as [$l_{(1-P)M/2}, l_{(1+P)M/2}$]. When it is symmetrical, the expanded uncertainty can be expressed as

$$U(l)=\frac{l_{(1+P)M/2}-l_{(1-P)M/2}}{2},$$

and the coverage factor is given by $k=U(l)/u(l)$.

4. Evaluation example

For easy to comparison, a standard gauge block, the nominal length as 50 mm, is measured on the MH3D-DCC-type CMM. Taking into account reducing the influence of the measurement strategy and the repeatability error of measurement on the measurement results, in the whole measurement process, the gauge block is placed on the CMM table fixed, and measuring coordinate system is selected to established on the gauge block, from its one surface of length direction are 10 points P_i (that $N=10$) taken up and down symmetrically, and P_k is taken from the center of the other surface of length direction, the number of automatic cycled measurement of CMM is set to 10 (that $m=10$). When calculating, the corresponding mean coordinates and arithmetic standard deviations are taken into. Using the mathematical model analyzed and built above, the measurement uncertainty of this task is evaluated. Since a standard gauge block used here, the influence on measurement result caused by its surface roughness can be ignored, the measurement model of the gauge block length is simplified from Eq.（3） as

$$l=L\cdot(1+\alpha_w\cdot\delta\theta)\cdot(1-\alpha_s\cdot\delta\theta)+\delta l_{res}+\delta l_v \quad (6)$$

The combined standard uncertainty of length measurement of the gauge block is simplified from Eq.（4） as, respectively,

$$u_c(l)=\sqrt{\left(\frac{\partial f}{\partial L}\right)^2\cdot u^2(L)+\left(\frac{\partial f}{\partial\delta\theta}\right)^2\cdot u^2(\delta\theta)+u^2(\delta l_{res})+u^2(\delta l_v)} \quad (7)$$

Where α_s = 1.05×10^{-6}/℃, α_w = 1.15×10^{-6}/℃.

The standard temperature of the CMM measurement is at 20℃, and the actual temperature of measurement varies between 19℃ and 21℃, meeting rectangular distribution, taken k as $\sqrt{3}$, measurement uncertainty caused by temperature error is given as $u(\delta\theta)=\frac{1}{\sqrt{3}}=0.57735$ ℃.

The maximum allowable indication error of the CMM is $3.0+4.0\times L/1000\mu m$, and the requested deviation of the gauge block should not exceed the amount of $\pm 0.07\mu m$, so the influence of indication error should not exceed the amount of $\pm 3.00028\mu m$, meeting rectangular distribution, taken k as $\sqrt{3}$, measurement uncertainty caused by indication error is given as $u(\delta l_{res})=\frac{3.00028}{\sqrt{3}}=1.73221\mu m$.

According to experience, to 0 level gauge block, the influence that the actual measurement point P_k deviating from the center of measuring surface take to measurement results is in range of $\pm 10nm$, meeting rectangular distribution, taken k as $\sqrt{3}$, so measurement uncertainty caused by eccentricity measurement is given as $u(\delta l_v)=\frac{10}{\sqrt{3}}=5.7735nm$.

According to the measuring points P_i and P_k, and Eq. (1),(2),(4),(5),(6) and (7), the evaluation results of the gauge block length based on GUM method can be obtained.

To further verify the feasibility of the mathematical model built and its corresponding uncertainty evaluation results, Monte Carlo simulation method is used again, sample size M was taken as 100 000, M random numbers corresponding with the input quantities are generated through the use of LabVIEW software according to the distributions of the input quantities (as Table 1 shows, the coordinate Measuring inputs meeting normal distribution[3]), then they are taken into Eq. (1), (2) and (6) , M samples of the gauge block length are gotten after calculating, finally, the corresponding uncertainty evaluation results are obtained. The results obtained respectively from MCM and from GUM method are compared as Table 2

Seen from Table 2, the evaluation results obtained both from GUM method and from MCM method meet the required precision of calibrated dimension of the standard gauge block, it shows that the mathematical model of measurement established above is feasible. Moreover, it also can be seen that the evaluation results obtained from the two methods are inconsistent by comparison, to theoretical analysis is concerned, the evaluation results obtained from MCM should be more reliable.

Input	Parameters		Input	Parameters	
	μ	σ		μ	σ
x_k /mm	-49.99625	0.00009	x_6 /mm	-0.00336	0.00018
y_k /mm	17.04351	0.01269	y_6 /mm	6.99783	0.00002
z_k /mm	-4.74436	0.00472	z_6 /mm	-5.47583	0.00205
x_1 /mm	0.00890	0.00008	x_7 /mm	-0.00120	0.00016
y_1 /mm	30.08028	0.00389	y_7 /mm	11.44734	0.00169
z_1 /mm	-0.97094	0.00004	z_7 /mm	-5.44656	0.00383
x_2 /mm	0.00793	0.00006	x_8 /mm	0.00216	0.00003
y_2 /mm	23.99441	0.00146	y_8 /mm	16.76682	0.00004
z_2 /mm	-0.98324	0.00003	z_8 /mm	-5.45270	0.00204
x_3 /mm	0.00496	0.00011	x_9 /mm	0.00299	0.00009
y_3 /mm	17.88451	0.00888	y_9 /mm	22.72968	0.00018
z_3 /mm	-1.01773	0.00461	z_9 /mm	-5.45933	0.00002
x_4 /mm	0.00351	0.00015	x_{10} /mm	0.00529	0.00007
y_4 /mm	10.93734	0.00218	y_{10} /mm	29.31215	0.00014
z_4 /mm	-0.99970	0.00410	z_{10} /mm	-5.45684	0.00326
x_5 /mm	0.00255	0.00008			
y_5 /mm	7.05158	0.01248			
z_5 /mm	-0.98844	0.00042			

Table 1 The distribution parameters of the input quantities

Method	l /mm	$u(l)$	k	U_{95}	Coverage interval meeting P = 95%
GUM	50.00086	0.03063	2	0.06126	[49.93960, 50.06212]
MCM	49.99777	0.00177	1.68	0.00296	[49.99481, 50.00073]

Table 2 The comparison of the results obtained by MCM method and GUM method

5. Conclusion

This paper presents an uncertainty evaluation example of the gauge block measurement, and verifies that the general mathematical model of geometry length measurement of CMM is suitable for general geometry length measurement, especially for short distance measurement, and has some practical value. The uncertainty evaluation of length measurement of the gauge block has given out the complete results and enhanced the value of CMM. Moreover, seen from the evaluation example, Monte Carlo method for uncertainty evaluation of complex measurement models is easy and efficient, so it has practical value.

In addition, it can be seen that the evaluation results obtained from GUM method and from MCM method are inconsistent, therefore, it is still a subject to be studied that how to verify the evaluation results and how to verify their reliability and evaluation accuracy through theoretical analysis and experimental verification.

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