A method of identifying and correcting the bias error of sensor using elliptic standard in the roundness measurement

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In high precision roundness measurement, only the eccentric error is taken into account in the traditional method, but actually the bias error of sensor probe also causes great effect to the measurement result, which is hard to find, and is hard to separate in single measurement. The paper provides a method of identifying and correcting the bias error of sensor using the elliptic standard, which is on the basis of the invariant direction of elliptic standard long axis, so it can calibrate the direction and value of the bias error. The method is achieved by doing twice measurement in the radial direction of elliptic standard. Using harmonic analysis to the system error and doing error analysis to the evaluation value of bias error, certificates the reasonability of the method in the paper completely, and also certifies that the method provides further evidence to increase the roundness measurement precision.

1. Introduction

In high precision roundness measurement, the hypothesis of real measurement condition based on the traditional model of Limacon [1] could not meet the challenge, because the effect caused by the bias error [2] of sensor probe is not taken into account. Bias error is caused by deviation between the inductive direction of sensor probe and the rotation axis of measurement. A typical roundness measurement model [3] is illustrated in Fig.1, which contains both the eccentric error and the bias error. Where e is eccentric value, α is bias angle, and d is the bias error of probe. O_1 is the rotation center of measurement, O_2 is the geometry center of object, O_3 is the instantaneous rotation center of the measurement line, R is radius of the zero value, r_0 is the least squares radius of objet. For the point of No. i, ρ_i is the distance between probe and O_3, r_i is the distance between probe and the geometry center. θ_i is the sample angle to the measure rotation center, and η_i is the sample angle to the geometry center. δ_i is the angle between the direction of measurement line and the direction of object surface profile.

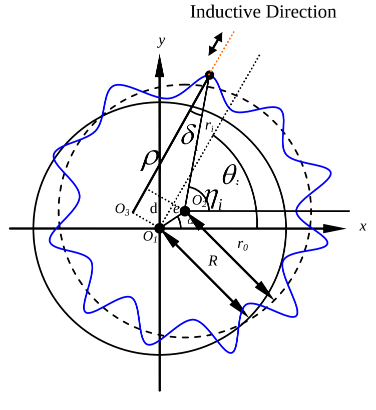

Fig.1 Model contains eccentric error and bias error

The measurement model deduces the measurement function as follows:

$$\rho_i = e\cos(\theta_i - \alpha) + \sqrt{r_i^2 - [d + e\sin(\theta_i - \alpha)]^2} \quad (1)$$

Where $i = 1, 2, \cdots, n$, and n is number of sampling.

Theoretically, for a group of measured data ρ_i, the nonlinear least squares method could be used to estimate the value of d, e, α at the same time. But because the function (1) possess the property of multi peek value, and the initial iterate value causes big effect to the final result of iterate convergence, it is easy to convergent to a false local minimum, which will produce a fault evaluation of bias error d. Against the issue above, the paper will discuss in detail on how to calibrate the bias error effectively with elliptic standard.

2. Analysis the measured signal constitution of elliptic standard

Using Taylor function to function (1), we can get:

$$\rho_i = e\cos(\theta_i-\alpha)+r_i-\frac{d^2}{2r_i}-\frac{de}{r_i}\sin(\theta_i-\alpha)-\frac{e^2}{2r_i}\sin^2(\theta_i-\alpha)+\cdots \tag{2}$$

The contour of elliptic standard approximates to be a model of secondary harmonics:

$$r_i = r_0 + A\cos 2(\eta_i-\varphi) \tag{3}$$

Then, its roundness value is $2A$, and direction angle of long axis is φ. If assuming that the preferences of elliptic standard used is, radius r_0=10mm, roundness value is 2μm, and d, the bias error of probe is -1mm~1mm. So the third item of function (2) can approximate to be:

$$\frac{d^2}{2[r_0+A\cos 2(\eta_i-\varphi)]}\approx\frac{d^2}{2r_0^2}[1-A\cos 2(\eta_i-\varphi)] \tag{4}$$

So as to the forth item:

$$\frac{de}{r_0+A\cos 2(\eta_i-\varphi)}\sin(\theta_i-\alpha)\approx\frac{de}{r_0}\sin(\theta_i-\alpha) \tag{5}$$

When the ratio of eccentric error and radius is $e/r_0\leq 10^{-3}$, the fifth item can approximate to be 0.

Based on the measurement model, the relationship of η_i and θ_i is:

$$\begin{cases}\delta_i=\eta_i-\theta_i\\ r_i\sin\delta_i=d+e\sin(\theta_i-\alpha)\end{cases} \tag{6}$$

So the expression of η_i based on θ_i can be obtained as:

$$\eta_i=\theta_i+\arcsin\left(\frac{d}{r_0}+\frac{e}{r_0}\sin(\theta_i-\alpha)\right) \tag{7}$$

Using Taylor function to function (7), we can get:

$$\eta_i\approx\theta_i+\arcsin\frac{d}{r_0}+\frac{e}{\sqrt{r_0^2-d^2}}\sin(\theta_i-\alpha)+\ldots \tag{8}$$

It is obvious that the relationship between η_i and θ_i approximates to be linear, and bias error causes an angle deviation between them, at the same time, eccentric error and bias error cause nonlinear fluctuation between angles relationship together. Under the condition $e/r_0\leq 10^{-3}$, we can get:

$$\eta_i\approx\theta_i+\arcsin\frac{d}{r_0} \tag{9}$$

So the measured signal constitution of elliptic standard, under condition of $e/r_0\leq 10^{-3}$, is shown as:

$$\Delta\rho_i=[e\cos(\theta_i-\alpha)-\frac{de}{r_0}\sin(\theta_i-\alpha)]+[(1+\frac{d^2}{2r_0^2})A\cos(2\theta_i+2\arcsin\frac{d}{r_0}-\varphi)] \tag{10}$$

In function (10), the measured signal $\Delta\rho_i$ can be divided into two parts: one is the first harmonic of signal, while another is the second harmonic.

If order the value of first harmonic to be a_1, phase to be ψ_1; the value of second harmonic to be a_2, phase to be ψ_2. So we can get:

$$e=a_1/\sqrt{1+\frac{d^2}{r_0^2}},\alpha=\psi_1-\arcsin\frac{d}{r_0} \tag{11}$$

$$P=2A=\frac{2a_2}{1+d^2/2r_0^2},\varphi=\psi_2/2-\arcsin\frac{d}{r_0} \tag{12}$$

If the method of harmonic analysis[4] is used to evaluate the value and angle of eccentric, roundness of elliptic standard and its long axis direction, that $\hat{e}=a_1$, $\hat{\alpha}=\psi_1$, $\hat{P}=2a_2$, $\hat{\varphi}=\psi_2/2$, which will cause system evaluation error:

$$\delta e=\frac{\hat{e}-e}{e}=\sqrt{1+\frac{d^2}{r_0^2}}-1,\delta\alpha=\hat{\alpha}-\alpha=\arcsin\frac{d}{r_0} \tag{13}$$

$$\delta P=\frac{\hat{P}-P}{P}=\frac{d^2}{r_0^2},\delta\varphi=\arcsin\frac{d}{r_0} \tag{14}$$

Its evaluation error is shown in the Fig.2 and Fig.3:

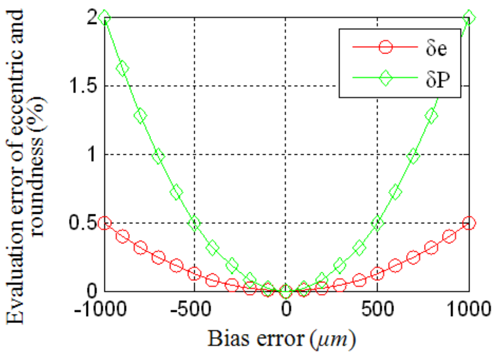

Fig.2 Evaluation error of eccentric and roundness

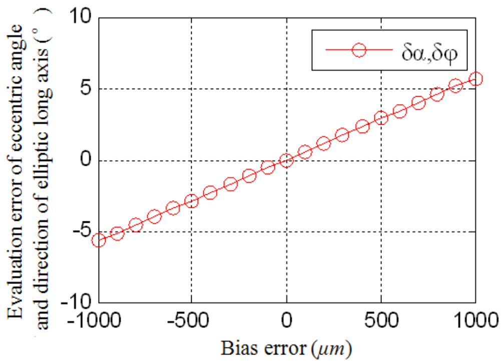

Fig.3 Evaluation error of eccentric angle and direction of elliptic long axis

The figures show that for the roundness contour only with secondary harmonic, when bias error is zero, the evaluation error is zero, that the first harmonic can be used to evaluate the value and angle of eccentric. The evaluation error will increase if the bias error increases. If the value of evaluation error is ordered to be less than 0.5%, the bias error should be less than 500μm; if the value of evaluation error is ordered to be less than 0.1%, the bias error should be less than 220μm. The similar result could be achieved by analyzing roundness contour with other harmonics. So, in order to obtain high precision roundness measurement result, it is necessary to calibrate bias error accurately, and to control it in the range of several hundred micrometers.

3. Principle of calibrating bias error with elliptic standard

For the roundness measurement instrument in Fig.4 and Fig.5, locate the elliptic standard on the work piece table, and drive the measurement arm to move the sensor to location (1). Adjust the elliptic standard to make the first harmonic value of signal around 0.1µm, which is to guarantee that the eccentric value is small enough, then obtains the first group of data ρ_i by sampling. Keep the location of elliptic standard unchangeable and drive the measurement arm again to location (2) which is on the opposite side of radial direction to location (1), then obtain data ρ_i' by sampling.

By doing harmonic analysis to data ρ_i and ρ_i', the value and phase of secondary harmonic can be achieved, shown as a_{12}, ψ_{12}, a_{22}, ψ_{22}. For the two measurements, if order the start point of f

First measurement sampling is the positive direction of x axis, the start point of second measurement sampling must be the negative direction of x axis, so their bias error are the same, but in opposite direction.

$$\psi_{22} - \psi_{12} - 2\arcsin\frac{d}{r_0} = 0 \tag{15}$$

Thus, the evaluation of bias error is:

$$\hat{d} = \sin(\frac{\psi_{22} - \psi_{12}}{2})r_0 \tag{16}$$

4. Error analysis of the bias error evaluation value

Based on the function $d = \sin(\frac{\psi_{22} - \psi_{12}}{2})r_0$ and linear superposition principle of error, the error is:

$$\delta d = \sqrt{r_0^2 - d^2}\,\delta(\frac{\psi_{22} - \psi_{12}}{2}) + \frac{d}{r_0}\delta r_0 \tag{17}$$

Where $\delta(\frac{\psi_{22} - \psi_{12}}{2})$ is error of measured angle, and it is

Sample points	2048	4096	8192	16384
Angle definition （°）	0.1758	0.0879	0.0439	0.0220

Table.1 Relationship between angle definition and account of sample data

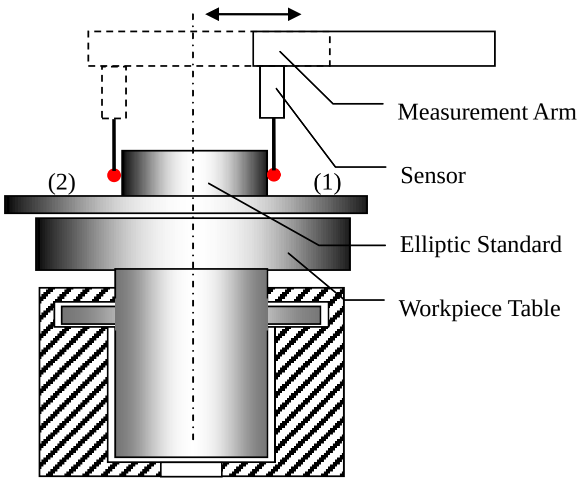

Fig.4 Instrument of roundness measurement model

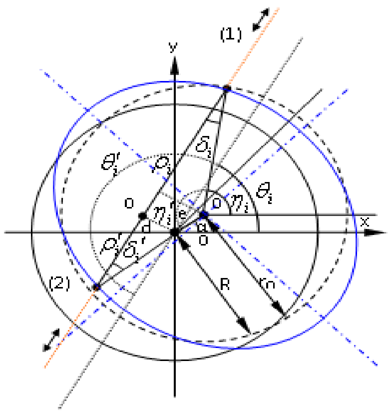

Fig.5 Principle of calibrating bias error using elliptic standard

On the basis of function (12), the direction of elliptic long axis in the two measurements are $\varphi_1 = \psi_{12} + \arcsin d / r_0$, $\varphi_2 = \psi_{22} - \arcsin d / r_0$. For the start points of two measurements are different by 180°, so both of their long axis direction and the direction angle of sampling start point are the same.

calculated by angle definition. For the roundness and cylinder measurement instrument, their angle definition is decided by amount of sampling, and their relationship is shown in Table 1.

δr_0 is the measurement error of elliptic standard radius, and it will be no less than its roundness, commonly around 1~2µm. If assuming that the radius of elliptic standard r_0=10mm, d is -1mm~1mm, Fig.6 and Fig.7 is achieved:

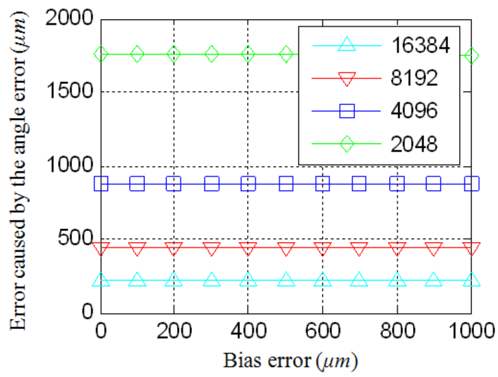

Fig.6 Bias error caused by the angle error

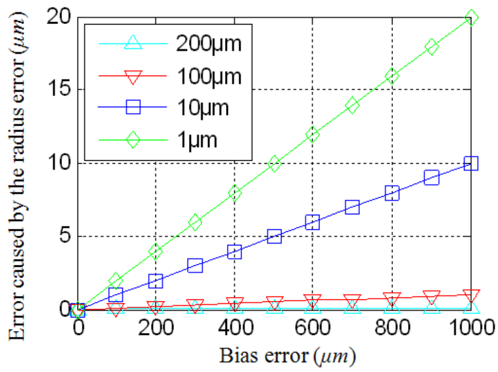

Fig.7 Bias error caused by the radius error

So, the measurement error caused by angle error occupies the main part, and when the account of sampling is 16384, the measurement error would be 220μm. And under the condition that the radius error of elliptic standard is 200μm, the caused measurement error of bias error is only 20nm, which is far more less than the one caused by angle error, thus square caliper or micrometer calipers are available enough to measure the radius of elliptic standard.

5. Conclusions

The paper realizes evaluating the bias error of sensor probe, by doing twice measurement in the radial direction of elliptic standard, on the basis of the invariant direction of elliptic standard long axis. Its measurement definition is direct ratio to account of sampling points, and is inverse ratio to radius of elliptic standard. When the account of sampling points are 16384 and the radius of elliptic standard r_0=10mm, the measurement definition is 220μm. Higher precision measurement result of bias error could be achieved by increasing sampling points or using elliptic standard with smaller radius. For roundness or cylinder measurement instrument, it has already met the challenge of measurement precision by controlling the bias error in the range of several hundred micrometers. So the method in the paper provides a reliable tool for calculating and modifying the bias error, and it is surely the evidence for further increasing roundness measurement precision.

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