Estimation of probe position for portable 3D vision coordinate measuring system

LIU Shugui, TANG Shaliang and DONG Yinghua

92 Weijin Rd., Naikai District, Tianjin University, Tianjin 300072, P.R. China
State Key Laboratory of Precision Measuring Technology and Instruments , Tianjin University
sgliu@tju.edu.cn, TEL: +86-13612103881, FAX: +86-22-2740-4778

KEYWORDS : Probe tip position, Standard cone, Least square fitting, Auto recognition

In this paper, a new approach to the position estimation of the probe tip which is fixed on a portable light pen is proposed. The light pen whose probe tip can be changed is employed in a 3D vision coordinate measuring system. More than 100 images of the light pen with different postures are collected, while its probe tip is constantly kept in a standard conical hole. By computing the rotation and translation matrixes in accordance with the properties of object and image, the probe tip position is obtained by the least square fitting method. The accuracy of the mathematical model and the effectiveness of the algorithm are proved. This approach can assist the auto recognition of the probe tip being installed.

Manuscript received: January XX, 2011 / Accepted: January XX, 2011

1. Introduction

3D coordinate measuring machines are widely used in industries such as machine manufacturing, automobiles and aviation for the advantages of wide measuring range, high accuracy etc[1]. However, the current coordinate measuring machines cannot meet the needs of quickly measuring huge parts in the field.

In recent years, various types of portable 3D vision coordinate measuring systems have been developed [2] [3]. For example, T-Pro made by Lecia, ProCam made by Accurex, solo/duo made by Metronor and Actiris made by Acticm. All these systems own the advantages of light and flexible, because is only the image of control points, not the image of the probe tip, needed to determine the coordinates of the point been measured. Moreover, the measuring precision is not affected by the quality, such as rigidity and curvature, of the surface being measured.

Under certain circumstances, especially when measuring large work piece with small holes, hidden points or corners, different probes are needed to complete the measuring process. During such process, the CCD camera and the work piece cannot be moved. And there is always no other proper machines to calibrate the position of the probe tip at present. If inaccurate information of the probe is adopted, the result of the measurement would be totally wrong. In this paper, a new approach to give the estimation of the probe tip's position when the probe is changed to meet the measuring requirement is introduced. This approach can also assist the auto recognition of the probe in use.

2. System Setup and Process

The portable 3D vision coordinate measuring system consists of a light pen, a CCD camera, and a computer, as shown in Figure 1.

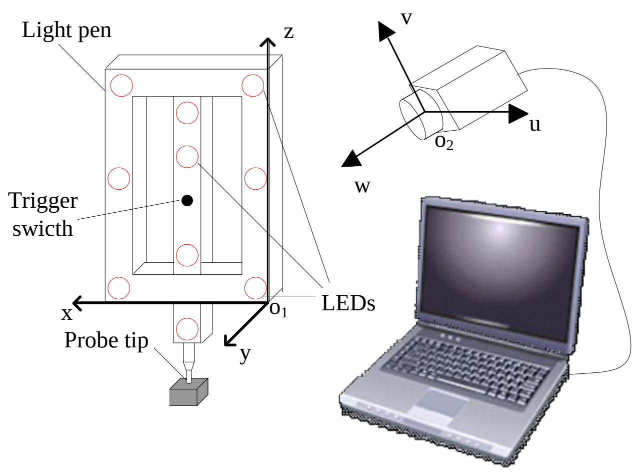

Fig.1 The portable 3D vision coordinate measuring system

On the light pen, there are 10 LEDs, whose distances between each other are already known. LEDs are on with pressing the trigger switch and give signals to the CCD camera to take images of the light pen. The probe can be changed to make the measurement more convenient.

When measuring, the probe tip is brought into contact with the point being measured. Receiving the signal given by the trigger switch, an image of the light pen with 10 LEDs on is taken by CCD camera and transferred to the computer. The software system installed in the computer then processes the image to obtain the coordinates of the center of the 10 LEDs on the image plane, which are used as the input to calculate the output of the system, the coordinates of the probe tip center in the world coordinate system (the coordinates of the point being measured). The measuring process is shown in Fig.2.

```mermaid
flowchart TD
    A[Detection Point] --> B[CCD Camera]
    B --> C[Image Acqusition Board]
    C --> D[Software Process System]
    D --> E[Coordinates]
```

Fig.2 System measuring process

3. Mathematical Model for the Probe Position

3.1 Introduction of the coordinate systems used

In order to build the mathematical model for the probe tip position, 3 coordinate systems are introduced[4], as shown in Fig. 1.

a) The light pen coordinate system $o_1 - xyz$: using the plane of the light pen as plane xz. y Axis is defined according to the right-handed system. The probe tip position we have to get is the very coordinates of the probe tip center under $o_1 - xyz$.

b) The camera coordinate system $o_2 - uvw$: using the perspective center as the origin of coordinates. The axis from the perspective center pointing to the crossing point of the optic axis and the image plane is defined as the w axis. u and v axis are parallel to the horizontal and vertical directions of the pixels of the CCD camera, respectively. The relationship between $o_2 - uvw$ and $o_1 - xyz$ can be described by the rotation matrix R and the translational matrix T:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} R & T \end{bmatrix} \bullet \begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix} \quad (1)$$

where, $R = \begin{pmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{pmatrix} \qquad \boldsymbol{T} = \begin{pmatrix} \boldsymbol{t}_1 \\ \boldsymbol{t}_2 \\ \boldsymbol{t}_3 \end{pmatrix}$

c) The world coordinate system $o_3 - x'y'z'$: in order to make the problem less complex, the world coordinate system is defined the same as the camera coordinate system $o_2 - uvw$.

3.2 Computation of the probe tip position

Build the portable 3D vision coordinate measuring system and bring the probe tip into contact with the standard cone which is fixed on a vibration isolation table in front of the CCD camera. Adjust the position of the CCD camera and fix the camera on a vibration isolation table to ensure that the control points (LEDs) of the light pen appear fully in the field of view when the rotation range of the light pen is large. Then, as shown in Fig.3, keep the probe tip on the standard cone while swinging the light pen smoothly. Then many images of the light pen with various orientations can be taken. Insist that the ideal coordinates of the probe tip center under $o_1 - xyz$ and $o_2 - uvw$ are $(x, y, z)^T$ and $(u, v, w)^T$ respectively. As the light pen is a rigid body, $(x, y, z)^T$ can be thought to be a constant. Because the position of the CCD camera is changeless, $(u, v, w)^T$ can also be considered as a constant.

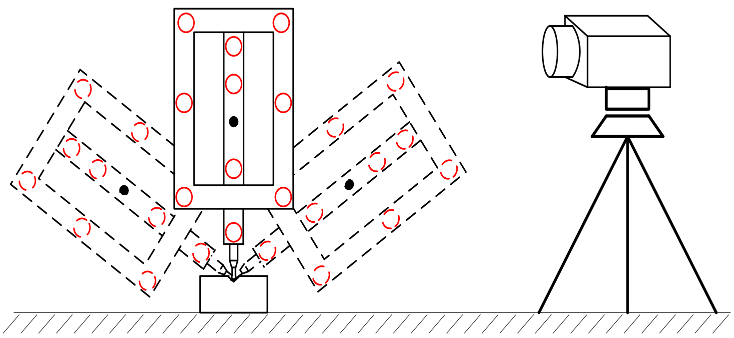

Fig.3 Image collection

Insist that the measured coordinates of the probe tip center with image i under $o_2 - uvw$ is $(u_i, v_i, w_i)^T$, according to equation (1), the following equation can be obtained:

$$\begin{pmatrix} u_i \\ v_i \\ w_i \\ 1 \end{pmatrix} = \begin{pmatrix} r_{1i} & r_{2i} & r_{3i} & t_{1i} \\ r_{4i} & r_{5i} & r_{6i} & t_{2i} \\ r_{7i} & r_{8i} & r_{9i} & t_{3i} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} r_{1i}x + r_{2i}y + r_{3i}z + t_{1i} \\ r_{4i}x + r_{5i}y + r_{6i}z + t_{2i} \\ r_{7i}x + r_{8i}y + r_{9i}z + t_{3i} \\ 1 \end{pmatrix} \quad (2)$$

where $\begin{pmatrix} r_{1i} & r_{2i} & r_{3i} \\ r_{4i} & r_{5i} & r_{6i} \\ r_{7i} & r_{8i} & r_{9i} \end{pmatrix}$ and $\begin{pmatrix} t_{1i} \\ t_{2i} \\ t_{3i} \end{pmatrix}$ can be obtained by the Gauss-Newton iteration method[5].

The deviation of the measured coordinate of the probe tip center under o_2-uvw from the ideal one can be presented as follows:

$$f_i\left(x,y,z,u,v,w\right)=\sqrt{\left(u_i-u\right)^2+\left(v_i-v\right)^2+\left(w_i-w\right)^2} \tag{3}$$

Based on the least squares method, only when the sum of the square of the deviations above reaches the minimum can we make certain of the probe tip position. Then the objective function of this optimization problem could be given as follows:

$$g(X)=\sum_{i=1}^{n}(f_i)^2 \tag{4}$$

where the optimization variable X represents 6 variables: $\left(x,y,z,u,v,w\right)$.

To deal with (4), general inverse method for least square solution of nonlinear multivariable equations is employed.

4. Design of the algorithm

4.1 General inverse method for least square solution to the system of nonlinear equations

Insisting that the system of nonlinear equations is:

$$f_i\left(x,y,z,u,v,w\right)=0 \tag{5}$$

then the Jacobi matrix of the system of nonlinear equations is:

$$A=\begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} & \frac{\partial f_1}{\partial u} & \frac{\partial f_1}{\partial v} & \frac{\partial f_1}{\partial w} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} & \frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial v} & \frac{\partial f_2}{\partial w} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial x} & \frac{\partial f_n}{\partial y} & \frac{\partial f_n}{\partial z} & \frac{\partial f_n}{\partial u} & \frac{\partial f_n}{\partial v} & \frac{\partial f_n}{\partial w} \end{bmatrix} \tag{6}$$

The iterative formula for the least square solution to the system of nonlinear equations is:

$$X^{(k+1)}=X^{(k)}-\alpha_k\Delta X^{(k)} \tag{7}$$

where $\Delta X^{(k)}$ is the least square solution to the system of linear equations $A^{(k)}\Delta X^{(k)}=F^{(k)}$. $A^{(k)}$ is the Jacobi matrix of $X^{(k)}$; $F^{(k)}$ is the value of the left side of the function after k times of iteration and could be presented as follows:

$$F^{(k)}=(f_1^{(k)},f_2^{(k)},\cdots,f_n^{(k)})^T \tag{8}$$

$$f_i^{(k)}=f_i(x^{(k)},y^{(k)},z^{(k)},u^{(k)},v^{(k)},w^{(k)}), \qquad i=1,2,\cdots,n$$

where α_k was the value which enable the one-variable function $\sum_{i=1}^{n}(f_i^{(k+1)})^2$ about α reaching the minimum value.

4.2 The initial parameters of the iterative algorithm

The initial parameters must given are $\left(x,y,z\right)^T$ and $\left(u,v,w\right)^T$, the ideal coordinates of the probe tip center under o_1-xyz and o_2-uvw, respectively. Proper initial parameters can reduce the run time of the iterative algorithm. From equation (2), insisting that $\left(u_i,v_i,w_i\right)^T$, and the coordinates of the probe tip center been measured are exactly the same with $\left(u,v,w\right)^T$, the following equation can be obtained:

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix}=\begin{pmatrix} r_{1i} & r_{2i} & r_{3i} & t_{1i} \\ r_{4i} & r_{5i} & r_{6i} & t_{2i} \\ r_{7i} & r_{8i} & r_{9i} & t_{3i} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \tag{9}$$

Equation (9) can be transformed into:

$$\begin{pmatrix} 1 & 0 & 0 & -r_{1i} & -r_{2i} & -r_{3i} \\ 0 & 1 & 0 & -r_{4i} & -r_{5i} & -r_{6i} \\ 0 & 0 & 1 & -r_{7i} & -r_{8i} & -r_{9i} \end{pmatrix}\begin{pmatrix} u \\ v \\ w \\ x \\ y \\ z \end{pmatrix}=\begin{pmatrix} t_{1i} \\ t_{2i} \\ t_{3i} \end{pmatrix} \tag{10}$$

From equation (9), it is apparently that there are 6 variables. As with only 2 images, 6 equations can be got, the smallest number of images to obtain the initial parameters is 2. If there are more than two images, with the corresponding $\begin{pmatrix} r_{1i} & r_{2i} & r_{3i} \\ r_{4i} & r_{5i} & r_{6i} \\ r_{7i} & r_{8i} & r_{9i} \end{pmatrix}$ and $\begin{pmatrix} t_{1i} \\ t_{2i} \\ t_{3i} \end{pmatrix}$, the initial parameters can be computed. In order to make the initial parameters more proper, all the images already gotten are used to form over determined linear equations, the solution of which are set as initial parameters of the iterative algorithm.

4.3 Calculation steps

Step 1: Take images and compute the corresponding R and T.

Step 2: Designate and calculate of the 6 initial values of the optimization variable X. The set of the initial value of the iterative time is $k=0$.

Step 3: Compute the function value $F^{(k)}$ and the Jacobi matrix $A^{(k)}$.

Step 4: Based on the rational extremum principle, find the point α_k which makes the function $\sum_{i=1}^{n}(f_i^{(k+1)})^2$ about α reaching the minimum value.

Step 5: Update the optimization variable X, which means doing iterative calculation of $X^{(k+1)}=X^{(k)}-\alpha_k\Delta X^{(k)}$.

Step 6: Estimate the convergence by the increment of $|a_k \Delta X^{(k)}|$. When the increment of $|a_k \Delta X^{(k)}|$ is small enough, end the iterative calculation. Otherwise, let $k = k+1$ and jump to the step 2.

Step 7: The position $(x, y, z)^T$ obtained with step 6 is the very probe tip position under the light pen coordinate system $o_1 - xyz$

5. Experiment and result

An experiment on comparison of measuring the probe tip position with the method mentioned above and an accurate method is carried out. A high-precision probe tip position called measured probe tip position (MPTP) can be obtained with a three coordinate machine. The coordinates of the probe tip center under $o_2 - uvw$ were computed and stored in the computer. In our experiment, 176 images are collected with CCD camera. With the mathematical model and the algorithm presented in section 3 and 4, the probe tip position what is called estimated probe tip position (EPTP) is calculated. The MPTP with a three coordinates machine is $(75.084, -23.801, -115.737)^T$, while the EPTP we get using the above method is $(75.303, -23.783, -115.290)^T$. By comparing the difference between the two probe positions, it is obvious that the method presented is effective and useful.

Another experiment is measuring a point of the standard cone with the two method mentioned above. Fix the CCD camera and the standard cone and measure the point of the standard cone for 10 times with the MPTP and the EPTP, respectively. The value of the standard deviation of the measurements, which are shown in Table 1, can be considered as an indication of the measurement precision. The standard deviation calculated proves that the precision of the estimated position is as high as the one of measured position.

Axis / Mode	X	Y	Z
With MPTP	0.060	0.052	0.844
With EPTP	0.047	0.060	0.842

Table. 1 The standard deviation of the 10 measurements of the same point (unit: mm)

6. Conclusions

In this paper, the mathematical model for the estimation of the probe tip position of the portable 3D vision coordinate measuring system is presented. The algorithm for the solution of the mathematical model is validated to be effective with our experiments. The measuring precision of the system can reach to the same level as with the measured probe tip position using a three coordinate machine. So the whole measuring system can be applied in industry manufacturing with its obvious advantages of convenient, flexible, fast and high accuracy. More experiments including repeatability tests calibration of different probes with various shapes and real measuring experiments are going to be put into practice.

REFERENCES

1. J. A. Bosch, “Coordinate Measuring Machines and Systems.” Marcel Dekker Inc., New York, USA, 1995
2. LIU Shu-gui, PENG Kai, HUANG Feng-shan, “A portable 3D vision coordinate measurement system using a light pen.” Key Engineering Materials, v 295-296, p 331-336, 2005
3. LIU Shu-gui, PENG Kai, ZHANG HF, HUANG Feng-shan,, “The study of dual 3D coordinate vision measurement system using a special probe.” Proceedings of SPIE - The International Society for Optical Engineering, v6357 II, Sixth International Symposium on Instrumentation and Control Technology: Signal Analysis, Measurement Theory, Photo-Electronic Technology, and Artificial Intelligence, p 63574H , 2006
4. LIU Shu-gui, HUANG Feng-shan, PENG Kai, “The modeling of portable 3D vision coordinate measuring system.” Proceedings of SPIE - The International Society for Optical Engineering, Optical Design and Testing II , v 5638, PART 2, p 835-842, 2005
5. PENG Kai, ZHANG Xue-fei, LIU Shu-gui, HUANG Feng-shan, “The Research of Vision Coordinates Measurement System Using N-Point Probe,” Journal of Transduction Technology, vol.20(7), p1635-1638, 2007 (in Chinese)