Research on dynamical model of time grating CNC turntable based on time series

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In order to apply a time grating , a novel displacement sensor with new principles, to full closed loop numerical control turntable instead of optical grating as a angle detector, it is necessary to transform temporal information to spatial information with time-space transformation algorithm for time grating. To reduce the dynamic forecast error and achieve a high accuracy position feedback, this paper analysis the dynamic characteristic between multi-factors and time quantity for time grating CNC turntable based on time series. According to analysis on autocorrelation function and partial autocorrelation function, dynamical model of time grating motion can be established. The next measurement angle can be forecasted based on series of past measured angle with dynamic model. Therefore, original absolute signal sampled in equal time interval can be transformed to continuous incremental pulses for full closed loop numerical control turntable. In order to reduce dynamic forecast error, 1-step-ahead forecast method is adopted. In addition, actual output incremental continuous pulses represent differentials between forecast incremental angular displacement and forecast error of last time. In this way, forecast error can be corrected between neighbouring sampling periods, and cumulative errors can be eliminated. To conform the validity of the dynamic model, a dynamic experiment system is designed. Experiment results prove that the accuracy of incremental time grating signal can reach ±3″.

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1. Introduction

Numerical control turntables are usually employed to machining and angle measurement, and its measurement accuracy is one of important parameters which evaluate manufacturing level. In current market, digital driving technology and displacement sensors are adopt for full closed-loop CNC system to realize digital measurement and position feedback in real time. And the testing accuracy of detector determines the position accuracy of moving parts. Grating sensors and other traditional angular encoders are adopted as incremental angular detectors for current CNC system. However, high accuracy grating sensors or other traditional angular encoders cost too much. Furthermore, sometimes professional elastic joints and signal division card should be adopted to improve the measurement resolution. Otherwise, additional error sources such as installation error and transmission error, will be brought in to influence system accuracy.

Forecast methods are currently used for industrial machining as a novel technology, and forecast control theory are widely studied[1]. In addition, error theory and error correction methods are adopted for reducing dynamic forecast errors[2]. In CNC field, many researchers study how to improve contour machining accuracy by forecasting CNC machine tool machining error and then compensating the error[3].

The time grating is a displacement sensor with a novel principle which adopts time quantity as measurement standard to measure spatial displacement. And angular displacement is measured very high accurately without a conventional mechanical scale. Therefore, it avoids machining precise slots to reduce machining difficulty and reduce production cost. In addition, it has the characteristic of strong anti-dust and anti-interference capabilities and high degree of intelligence. Therefore, research on time grating displacement sensor instead of traditional displacement sensors applying to closed-loop CNC system as angular detector has much more superiority in accuracy, stability and production cost.

Dynamic measurement plays important roles in modern metrology, and with the development of science and technology, dynamic measurement with high accuracy has been becoming main trends. Dynamic measurement defines[4]as following: measuring instantaneous value over time. i.e. test devices are working during dynamic process. Dynamic measurement has the characteristic of space-time property, randomness, correlation and dynamic property. Dynamic modeling of CNC turntable is presented to realize full closed-loop numerical control system with original absolute time grating displacement sensor as position detector.

2. Principle of forecast method

According to the measurement principle of time grating[5-7], rotating magnetic field is used to construct a moving coordinate system S' with constant velocity V, Spatial angular displacement θ between fixed probe P_b and movable probe P_a with random velocity v can be represented by scanning time difference $\triangle T$ between fixed probe and movable probe, i.e. θ=(360°/measurement period)×$\triangle T$. The time interval for the magnetic field of traveling wave scanning fixed probe is a constant which can be called as measurement period T as shown in Fig.1. The rising edge of fixed probe signal can be employed as trigger signal for data acquisition,

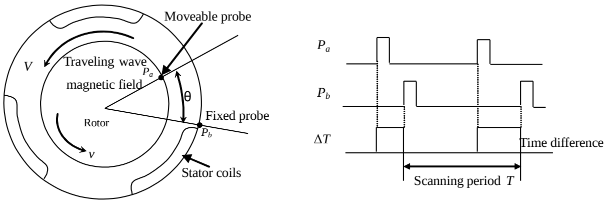

Fig.1 The measurement principle of time grating

and failing edge can be employed as stopping signal for data acquisition. So each measurement displacement value θ can be obtained every measurement period T. i.e. The detected angle by time grating sensor does not change during measurement period. Series of measured displacement value can be denoted as: $\theta_1 \ldots \theta_{i-n-1}$, $\theta_{i-n}, \ldots \theta_{i-1}$, θ_i, and corresponding sampling time points can be denoted as:$T_1 \ldots T_{i-n-1}$, $T_{i-n}, \ldots T_{i-1}$, T_i. Because of independent clock systems and different sampling period for time grating sensor and numerical control system, measured displacement value for both time grating sensor and numerical control system are not synchronized. As shown in Fig.2, sampling time points of time grating sensor T_{i-1}, T_i are not synchronized with that of numerical control system T_{CNC}. So the measured displacement value for numerical control system is not what is the turntable current position, but the position before Δt_{i-1}. When CNC turntable moves at high velocity, measurement delay will cause great feedback error which lead to failure.

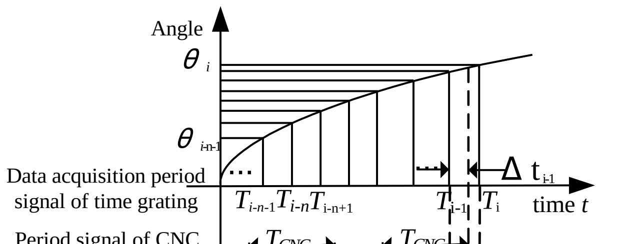

Fig.2 The principle of forecast method

In order to solve aforementioned asynchronized data acquisition, dynamic model should be established to describe the relationship between angular displacements of time grating CNC turntable and time. i.e. original absolute displacement signal sampled in equal time interval for time grating sensor should be transformed into continual incremental pulses. Using established dynamic measuring model, relationship between current output angle and series of past output angle can be obtained.

3. Principle of dynamic model

There are two ways to establish dynamic model[2]. One is pre-modeling before measurement. The other is modeling based on data processing after measurement. This research adopted the later method to establish dynamic model.

Suppose series of measured angular displacement: $\theta_1 \ldots \theta_{i-n-1}$, $\theta_{i-n}, \ldots \theta_{i-1}$, and corresponding sampling time points:$T_1 \ldots T_{i-n-1}$, $T_{i-n}, \ldots T_{i-1}$. Because series of measured angular displacement are obtained every measurement period T, measured angular displacements can be regarded as time series. According to the analysis on autocorrelation function and partial autocorrelation function with time series theory, the relationship between current measured angular displacements and past series of measured angular displacement can be obtained. Therefore, when the sample time point T_{i-1} comes, angular displacement $\hat{\theta}_i$ between T_{i-1} and T_i can be forecasted. Then the forecast incremental angular displacements $\Delta\hat{\theta}_i (\Delta\hat{\theta}_i = \hat{\theta}_i - \theta_{i-1})$ are transformed into continuous pulses to send out during measurement period T (between T_{i-1} and T_i),as shown in Fig.2. In this way, original absolute displacement signal sampled in equal time interval for time grating sensor can be transformed into continual incremental pulses. And the number of output pulses can be obtained by Equ.1:

$$P_i = (\Delta\hat{\theta}_i - e_{i-1}) / \mathrm{Q} \qquad (1)$$

Where Q is pulse equivalent, e_{i-1} is forecast error of last measurement period, i.e. residual error between measured value θ_{i-1} and forecasted value $\Delta\hat{\theta}_{i-1}$. 1-step-ahead forecast method[8-9] which employs current real measured value instead of its forecast value as sample data to ensure forecasting efficiency and accuracy. In this way, forecast error does not accumulate.

4. Methods

Time series analysis has been widely applied to many fields such as economy, industry and so on. And many achievements[10-13] have been achieved. A fundamental task of time series is to unveil the probability law that governs the observed time series. With such a probability law, the underlying dynamics can be understood, future events can be forecasted, and future events via intervention can be controlled.

The foundation of classic time series analysis is stationarity[8-9]. Strict stationarity requires that the joint distribution of $(\mathrm{X}_{t_1}, \cdots, \mathrm{X}_{t_k})$ is identical to that of $(\mathrm{X}_{t_1+t}, \cdots, \mathrm{X}_{t_k+t})$ under time shift. This is a very strong condition that is hard to verify empirically. Therefore, a weaker version of stationarity is often assumed. A time series $\{\mathrm{X}_t\}$ is weakly stationary if both the mean of X_t and the covariance between X_t and $\mathrm{X}_{t-\ell}$ are time-invariant, where ℓ is an arbitrary integer. In order to obtain series of stationary signal, the trend and the period should be extracted from original series of data. In this way, nonstationary signals can be transformed into stationary signals. Then the correlation between next measurement angle and series of past displacement measurement angle can be obtained with time series theory. The analysis method is detailed as following:

(1) Data preprocessing for series of measured angle

Suppose the series of original measured angle are $\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3 \ldots \mathrm{X}_t$. And after second-order difference process, the original measured value can be converted into $\mathrm{X}_1', \mathrm{X}_2', \mathrm{X}_3' \ldots \mathrm{X}_{t-2}'$.

Each data is normalized to converted into stationary series { X_t }with mean value 0：

$$\overline{x'} = \frac{1}{N}\sum_{j=1}^{N} x'_j \qquad (2)$$
$$X_t = x'_t - \overline{x'}, t = 1,2,\cdots,N$$

(2) Parameters estimation:

For time series $\{X_t\}$, relationships among the random variables at different time points t are interested. The autocorrelation function measures the dependence between X_{t+k} and X_t. The partial autocor - relation function is the correlation between the residual of X_{t+k} and that of X_t after regressing both linearly on $X_{t+1}, \cdots, X_{t+k-1}$.

Sample autocovariance $\hat{\gamma}_k$ and sample autocorrelation function (ACF) $\hat{\rho}_k$ are estimated as following:

$$\hat{\gamma}_k = \frac{1}{N}\sum_{j=1}^{N-k} X_j X_{j+k}, k = 0,1,\cdots,p \qquad (3)$$
$$\hat{\rho}_k = \hat{\gamma}_k \big/ \hat{\gamma}_0 \ .$$

Partial autocorrelation function (PACF) $\hat{\varphi}_k$ are estimated with Yule-Walker equation：

$$\begin{bmatrix} \hat{\varphi}_{p1} \\ \hat{\varphi}_{p2} \\ \vdots \\ \hat{\varphi}_{pp} \end{bmatrix} = \begin{bmatrix} 1 & \hat{\rho}_1 & \cdots & \hat{\rho}_{p-1} \\ \hat{\rho}_1 & 1 & \cdots & \hat{\rho}_{p-2} \\ \vdots & \vdots & \ldots & \vdots \\ \hat{\rho}_{p-1} & \hat{\rho}_{p-2} & \cdots & 1 \end{bmatrix}^{-1} \begin{bmatrix} \hat{\rho}_1 \\ \hat{\rho}_2 \\ \vdots \\ \hat{\rho}_p \end{bmatrix} \qquad (4)$$

In practice, the partial autocorrelation function $\hat{\varphi}_k$ estimation is often carried out in terms of Levinson–Durbin algorithm:

$$\begin{cases} \hat{\varphi}_{11} = \hat{\rho}_1 \\ \hat{\varphi}_{k+1,k+1} = \left(\hat{\rho}_{k+1} - \sum_{j=1}^{k} \hat{\rho}_{k+1-j}\hat{\varphi}_{k,j}\right)\left(1 - \sum_{j=1}^{k} \hat{\rho}_j \hat{\varphi}_{k,j}\right) \\ \hat{\varphi}_{k+1,j} = \hat{\varphi}_{k,j} - \hat{\varphi}_{k+1,k+1}\hat{\varphi}_{k,k+1-j}, 1 \le j \le k, k \le p \end{cases} \qquad (5)$$

(3) Model selection:

The most popular class of linear time series models consists of autoregressive moving average (ARMA) models, including purely autoregressive (AR) and purely moving-average (MA) models as special cases.

In selecting a model, interpretability, simplicity, and feasibility play important roles. A selected model should reasonably reflect the physical law that governs the data. Everything else being equal, a simple model is usually preferable. The family of probability models should be reasonably large to include the underlying probability law that has generated the data but should not be so large that defining parameters can no longer be estimated with reasonably good accuracy.

According to aforementioned principle, the sample partial autocorrelation function $\hat{\varphi}_k$ of process cuts off at lag *7*. But, its sample autocorrelation function $\hat{\rho}_k$ will not have a clear cutoff to 0 as the lag increase (more information is introduced in experiment section). Analysis results mean that the next angle $\hat{\theta}_i$ can be forecasted based on past angles $\theta_{i-7}, \theta_{i-6}, \ldots \theta_{i-1}$ with AR model.

(4) Model checking criterion:

A fitted model must be examined carefully to check for possible model inadequacy. If the model is adequate, then the residual series should behave as a white noise. There are several information criteria available to determine the order p of model. All of them are likelihood based. Akaika Information Criterion(AIC)[8-9] is reduce to

$$AIC(p) = N \ln \hat{\sigma} + 2p \qquad (6)$$

Where P is order, N is sample size.

5. Experiment results

In order to verify the dynamic model validity, a dynamic measurement was designed. As shown in F.g.3, experiment devices consisted of circular time grating, turntable, ac servo motor, Heidenhain circular grating and Siemens numerical control system. Time grating displacement sensor and optical grating, and turntable should be installed in a concentric common centre line. Each of them is connected by elastic joint. Time grating is employed as angle detecting sensor. And optical grating ROD880 with 36000 line count and accuracy of ±1″ is adopted as measurement standard to examine the accuracy of forecasting continuous pulses. In order to improve the resolution of optical grating, interpolation and digitizing electronics IBV660B is applied for converting the ROD880 sinusoidal signals to TTL square-wave pulse as well as 400-fold subdivision. Consequently the measuring resolution of the optical grating can reach 0.09″. The Fuji servo motor (GYS751DC2-T2C) is controlled by Siemens numerical control system to drive turntable, optical grating and time grating to rotate synchronously. Therefore, during the measurement process, they always rotate at the same velocity. Subdivided optical pulses with IBV660B and forecasting continuous pulses for time grating are input two counters together, and these two counters can lock input pulses synchronously to realize comparison in real time.

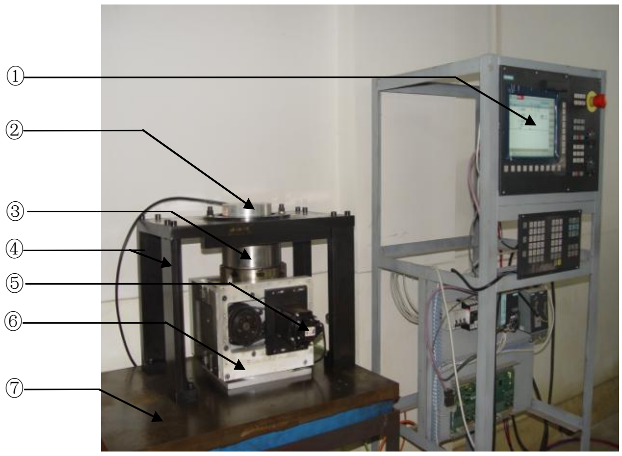

Fig.3. Experimental apparatus

Where,①Siemens 802D numerical control system.②Circular optical grating. ③Circular time grating displacement sensor. ④Stand column. ⑤Fuji ac servo motor (GYS751DC2-T2C).⑥Turntable. ⑦Worktable

Original measured displacement curve is illustrated in Fig.4. First -order difference curve is illustrated in Fig.5. It took 9.28s to rotate from 13.05740° to 55.41575° for time grating CNC turntable. Each angle displacement was sampled every 80 ms. And second-order difference curve is illustrated in Fig.6. Because measured angles are sampled in equal time interval, first order difference curve represents velocity variation and second order difference curve represents acceleration variation. From velocity curve, it can be seen that motion process experiences stationary state, acceleration state , motion with an approximate constant velocity, deceleration state and stationary state.

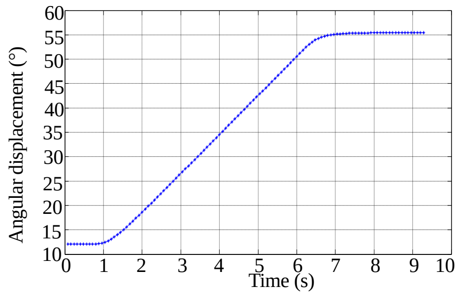

Fig.4 Measured angular displacement curve

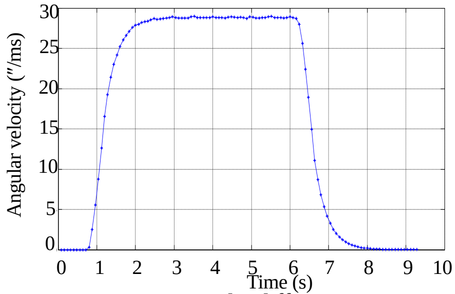

Fig.5 First order difference curve

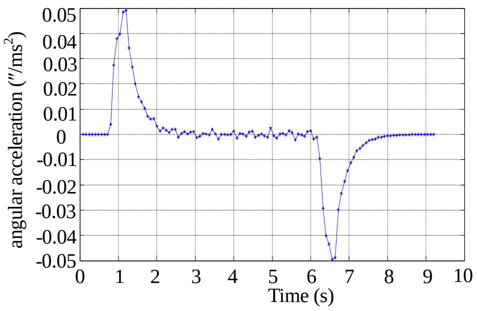

Fig.6 Second order difference curve

Plotting the estimated autocorrelation function and partial autocorrelation function against the time lag as shown in Fig.7 and Fig.8, is a useful technique in analyzing time series data. In autocorrelation function plots (ACF plots) and partial autocorrelation function plots (PACF plots), the horizontal lines (dashed lines in blue) at $\pm 1.96/\sqrt{N}$ are superimposed. These intervals give the pointwise acceptance region for testing the null hypothesis $H0 : \rho(k) = 0$ at the 5% significance level; They assist in judging whether a particular $\hat{\rho}_k$, $\hat{\varphi}_k$ is statistically significantly different from zero. According to aforementioned modeling principle, the sample PACF of process cuts off at lag *7*. But, its sample ACF will not have a clear cutoff due to random fluctuations. Therefore, dynamic model AR(7) can be determined by characteristic of autocorrelation function and partial autocorrelation function. And the dynamic model can be deduced as following:

$$X_t = 1.5456X_{t-1} - 0.7209X_{t-2} + 0.4506X_{t-3} - 0.5142X_{t-4} - 0.1258X_{t-5} + 0.4985X_{t-6} - 0.2042X_{t-7} + \varepsilon_t \quad (7)$$

Forecasting angular acceleration curve is illustrated in Fig.9, and forecast acceleration error range from -0.00046872″/ms^2 to 0.00043819″/ms^2. According to dynamic model, measured calibration error curve are within ±3″ as illustrated in Fig.10.

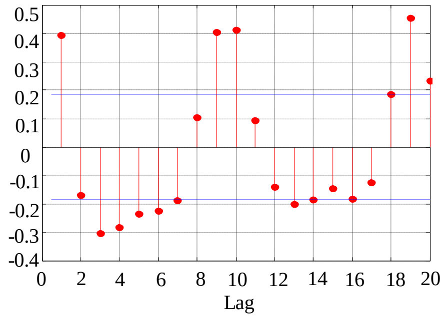

Fig.7. Autocorrelation function diagram

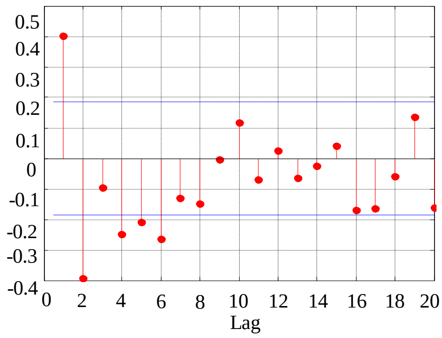

Fig.8 Partial autocorrelation function diagram

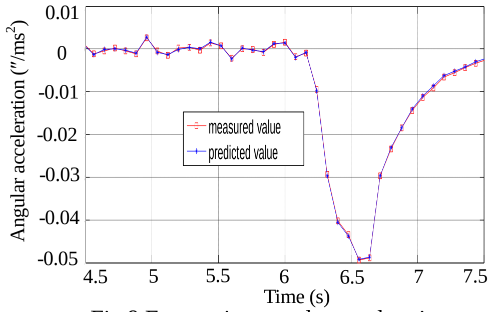

Fig.9 Forecasting angular acceleration curve

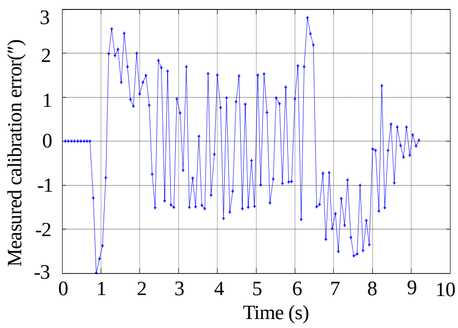

Fig.10 Measured calibration error curve

6. Discussions

(1) In order to satisfy the requirement of numerical control system, original absolute angle displacements are transformed into the continuous pulses with established dynamic model. Therefore, original absolute time grating displacement sensor can be employed as detecting angle sensors for full closed loop numerical system. In addition, the asynchronization of sampling period between time grating and numerical system can be solved.

(2) It can be seen from the Fig.10 that during abrupt acceleration or deceleration process, forecast errors are much more. On the contrary, if variation of acceleration is more stationary, forecast errors are much less.

7. Conclusions

Time series analysis is not just a sophisticated mathematical theory, its forecasting algorithm based on time-frequency data analysis has high accuracy. Difference algorithm is employed to convert nonstationary data into stationary data. Furthermore, more physical significance is given for pure mathematics difference algorithm. One order difference algorithm describes movement velocity of time grating, and two order difference algorithm describes movement acceleration.This paper introduced the forecast method for circular position of numerical control turntable based on time grating sensor. In this way, original absolute signal of time grating sampled in equal time interval can be transformed to incremental pulses. As to errors control, when current position forecast was conducted, a latest real measured angle of time grating sensor was interpolated as standard quantity to correct the last forecast error. Experiment results prove that the forecast error would not be cumulated and the dynamic error of turntable was within ±3″.

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