An integrated form error evaluation method for ultra-precision freeform surfaces

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Components with ultra-precision freeform surfaces have gained more and more applications in modern optical and measurement systems. To fulfill the unique requirements, freeform surfaces need to reach quite a critical form accuracy of sub-micrometer level. Now, there still lack of definite evaluation method for these surfaces. This paper develops an integrated method for ultra-precision freeform surface evaluation which generally includes two steps means initial-matching step and fine matching step. In initial-matching process, a dimension-reduced method is brought out. Such that the pre-matching can be high efficiently achieved by just matching of the feature points by using of point pattern recognition. In fine matching step, the Sequential quadratic programming SQP method was adopted to get the best transition and rotation parameters of the matching matrix. Form error can be obtained after the registration of the measured and the designed freeform surface by calculating the distance of corresponding points. Evaluation results show that this proposed method is capable of evaluating of ultra-precision freeform surfaces with high efficiency and accuracy.

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1. Introduction

Components with ultra-precision freeform surfaces have gained more and more applications like F-theta lens in laser printers [1], reflectors in automotive lighting [2], and sinusoidal grid surface in multi-axis measurement system [3]. To fulfill the unique optical or measurement requirements, freeform surfaces need to reach quite a critical form accuracy of sub-micrometer level. While, unlike traditional machined surfaces, no rotational centers, feature lines or holes can be found on freeform surfaces. Such that the fundamental difficulty for freeform research is how to measure and characterize the freeform surfaces with required accuracy [4]. The evaluation technique is quite important not only for the application, but also for the fabrication processing.

To obtain the form error, measured freeform surface needs to be rotated and translated such that it can be as close as possible with designed template. This process can be generalized as an optimization problem with optimize parameters of the six degree of freedoms. And the objective function can be least square or minimum zone criterion [5]. In the past years, many researchers have contributed to the freeform evaluation technique. One of the wildly used methods is the iterative closest point (ICP) method [6]. While, ICP process requires very good initial values of optimize parameters that was close enough to the true value such that it can converge to a global minimization [7]. To improve the matching accuracy, one method is to improve the optimal algorithms [8]; another is to provide good initial values [9]. While, the required evaluation accuracy of most previous objects just need to reach millimeter or sub-millimeter [10, 11]. And this is inadequate for the evaluation of ultra-precision freeform surfaces which are wildly used in modern optical components. Improper matching process or initial estimation of optimize parameters would lead to the failure of the evaluation process.

In recent years, some evaluation methods with high accuracy have been proposed. Coupled reference data method (CRDM) can evaluate freeform surfaces in nanometer scale precision, but additional reference features need to be manufactured together with the freeform surfaces [12]. This would increase the manufacturing cost greatly; and to some specific components it is even not allowed. To get relative good initial searching point, the two step matching strategy was developed including initial-matching and fine-matching. The initial matching methods include the five point pre-fixture [13] and structured region signature (SRS) method [14]. As to the five point pre-fixture method, measured area needs to be generally the same with the designed template. While, in most of the situations, measured area may be just a small portion of the designed surface. This would lead to large pre-matching error. Meanwhile, the five point pre-fixture method can not remove the rotation error around the normal vector. In the later proposed method, the authors innovatively registered the measured surface with the template using 2-D signature curve. This method can reach very high registration accuracy, but it needs to construct series of signature curves on the template and compare the curves with the signature curve of the measured surface. This method is complicated and very low efficient especially to a large area of freeform surface.

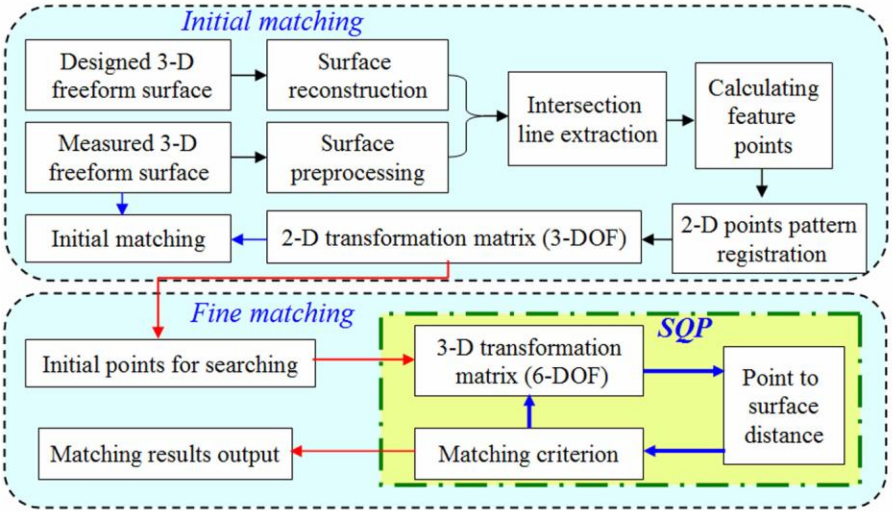


Fig.1 Framework of the freeform surface evaluation system

The aim of this paper is to develop an integrated method which can evaluate ultra-precision freeform surfaces with high efficiency and accuracy. It also includes two steps which are initial-matching step and fine-matching step. In initial-matching process, specific intersecting lines of both designed and measured surfaces are extracted as the feature lines which can represent the evaluated surfaces. To further improve the initial matching efficiency, points on the intersecting lines with maximum curvature are extracted and defined as the feature points, such that initial-matching can be achieved by just registering of these very limit feature points using point pattern matching technique. In fine-matching step, the SQP method was adopted to get the best transition and rotation parameters of the matching matrix. This method is high accurate and also quite efficient. Form error can be obtained after the registration of the measured and the designed freeform surface by calculating the distance of corresponding points. From the inspecting results of both simulated and measured freeform surfaces, it can be seen that this proposed method is capable of evaluating of ultra-precision freeform surfaces with high efficiency and accuracy.

2. Framework of freeform surface evaluation system

The framework of the integrated freeform surface evaluation system is shown by fig. 1 which is also achieved by two steps named initial-matching step and fine-matching step. Aim of the initial matching is to obtain good initial points for fine matching and the fine matching step is to get the optimal translation and rotation parameters fulfilling the matching criterion.

At the beginning of initial matching, both the description of designed 3-D freeform surface and the measured 3-D surface data are needed. The designed surface can be represented by mathematical functions or discrete data points. In the later case, designed data points need to be reconstructed so as to get the mathematical model. Surface preprocessing involves filtering and least square plane fitting. Filtering can separate the high frequency roughness surface from the measured result such that a smooth surface can be obtained. By using the least-square plane fitting technique, the relative consistent planes for both designed and measured surface can be constructed named the base planes. Then intersecting lines of both designed and measured freeform surfaces can be extracted. Feature points are defined as the ones with the maximum curvature on intersecting lines. Such that by registration of these very limit points the initial matching can be obtained and the 2-D transformation matrix can also be calculated.

Fine-matching process can be generalized as an optimization problem with the optimization parameters of the translation and rotation of the measured surface. Objective function is the matching criterion. In this proposed paper, the optimization problem is solved using SQP method and the 2-D transformation parameters can be used as the initial points for searching.

3. Development of the evaluation system

3.1 Problem statement of freeform surface matching

Designed surface is represented in the design frame while the measured points are in the measurement frame, as shown in fig. 2. The variance of designed and measured frame brings the difficulty for evaluating the form accuracy of measured surface. To obtain the form error, measured surface should be translated and rotated such that it can be well matched with the designed surface.

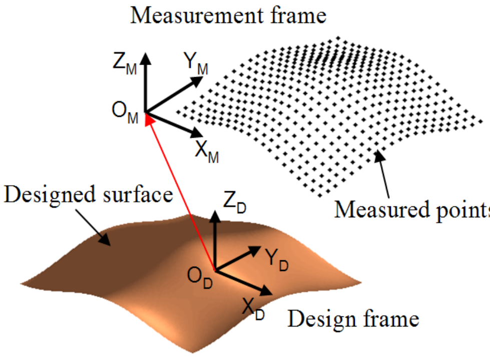


Fig.2 Illustration of freeform surface matching

Matching result can be evaluated by the matching criterions like average distance, average squared distance, and maximum distance deviation [5]. So the matching process can be generalized as an optimal problem to find out the translation and rotation parameters fulfilling the matching criterion. Usually, large difference exists between the ideal matched coordinate and the measured frame. In this situation, the solving of the optimal translation and rotation parameters comes out to be low efficient or non-convergent. Optimized parameters might be trapped at the local minimums. All of these would lead to the failure of the final evaluated results. To overcome these problems, the two steps matching method is also adopted in this presented paper.

3.2 Initial matching process

To achieve the initial matching efficiently and accurately, a dimension-reduced method is developed. Using this method, 3-D freeform surfaces can be matched by registration of several feature points. As shown by fig. 3, most of the time, measured points locate in space with free position. The first step is to find a least-square plane through the region of the measured surface (To simplify the figure, it was not drawn). This plane can be expressed in the formation of $ax+by+cz+1 = 0$, and the normal vector $\mathbf{n} = [a,b,c]^T$ can be calculated with an eigenvector method[15]. The base plane is defined as the one paralleling with the least-square plane and having a fixed distance to the specific point.

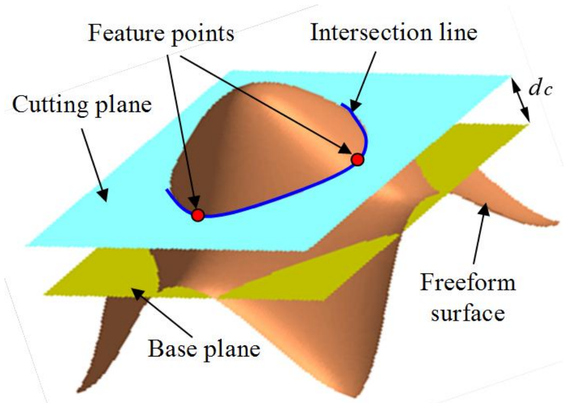


Fig.3 Illustration of the dimension-reduced method

This procedure is also done for designed surface, such that relatively unified base planes for each surface can be obtained. The cutting planes are those paralleling with each base plane and having the distances of d_c. The intersection line of freeform surface and the cutting plane can be considered as the feature line of the surface. Then we calculate the curvature of each point on the line and selected the ones with maximum curvature as the feature points.

Point pattern matching technique has been widely used in pattern recognition. In this proposed paper, a fast algorithm for point pattern matching is adopted which is of high efficient and robust [16]. Assuming $P = \{p_1, p_2,..., p_m\}$ is the designed feature points pattern and $Q =\{q_1, q_2,..., q_n\}$ is the measured feature points pattern. In 2-D coordinate system, each feature point can be described with the coordinate $(x,y)^T$. Such the designed and measured feature points can be represented as $P=\{(x_{p_i},y_{p_i})^T \mid i=1,2,\ldots,m\}$ and $Q=\{(x_{q_a},y_{q_a})^T \mid a=1,2,\ldots,n\}$ respectively. The first step of the point pattern registration is to find the corresponding point pairs of designed and measured point pattern. In actual situation, the measured surface could not be exactly the same with the designed surface due to various factors influencing the final form accuracy of freeform surface. Such that to the point pair (q_a, p_i), if $\| q_a q_b |-| p_i p_j \| \le d_e$, the point pair (q_b, p_j) can be considered as the supporting pair, where d_e is the allowable distortion bound. The relative angle of supporting point which is defined as: $\theta = \theta_{\overline{q_a q_b}} - \theta_{\overline{q_i q_j}}$ can also be calculated. The first matching point pair is the one with the maximum points, and the angle is maximum relative angle. Then other matching pairs between P and Q can be determined by calculating the distance and angle relative to the first matching pair. After finding all the matching pairs, a registration $G_{(t_x,t_y,\theta)}$ that could make this corresponding pair consistent can be obtained by using of a least-square estimation method. Where, θ is the rotated angle, t_x and t_y are the translation along x and y directions, respectively.

3.3 Fine matching process

Measured freeform surface is usually represented by 3-D desecrate points as $M = \{\mathbf{m}_k \in R^{3\times1} | k = 1, 2,\ldots, s\}$.The corresponding points to the measured points on the designed surface can be expressed as $N = \{\mathbf{n}_k \in R^{3\times1} | k = 1, 2,\ldots, s\}$. Assuming d_k is the distance between the kth measured and its corresponding point on the designed surface, the least-square matching criterion of average square distance can described as:

$$E = \sum_{k=1}^{s} \|d_k\|^2 \tag{1}$$

Let the designed surface keep stationary. With the translation and rotation of the measured surface, the relative position of the surfaces would be changing which also induces to the changing of the average square distance. The fine matching process can be generalized as an optimization problem with the following objective function.

$$\min E(X) = \sum_{k=1}^{s} \|\mathbf{n}_k - (\mathbf{R}\mathbf{m}_k + \mathbf{t})\|^2 \tag{2}$$

Where $\mathbf{R}$ is the rotation matrix, $\mathbf{t}$ is the translation vector and $\boldsymbol{X}$ is the translation and rotation vector: $\boldsymbol{X}= [T_x\ T_y\ T_z\ \theta_x\ \theta_y\ \theta_z]$. Such the optimization problem is to find the translation and rotation parameters to fulfill the objective function. The calculation of the corresponding point on the designed surface is the initial step for solving of the optimization problem. The corresponding point can be defined as the one on designed surface which has the minimum distance to the specific measured point.

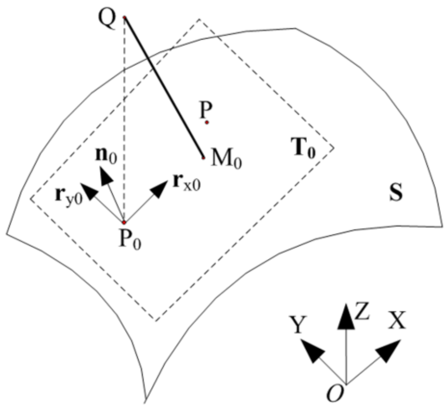


Fig.4 Calculating point to surface distance

An iteration method is developed for the calculation. The first iteration step is illustrated by figure 4, where Q is a measured point, P_0 is the initial iteration point, P is the corresponding point, **S** is the designed surface, T_0 is a tangent plane which is constructed by the coordinate of P_0 and its tangential vector $\mathbf{r}_{x0}$ and $\mathbf{r}_{y0}$. Assuming M_0 is the projection point of Q on T_0, its x and y coordinate can be obtained by solving the straight line equation and the tangent plane, for the first iteration, they are x_1 and y_1, respectively. A new point P_1 with the coordinate of $(x_1, y_1, S(x_1, y_1))$ can be considered the start point for the next iteration. For each iteration, the distance between Q and the lth iteration point P_l is also calculated. If $\|Q P_l| - |Q P_{l-1}\| < \varepsilon$, P_l can be considered as the corresponding point of Q, where ε is a given allowed tolerance.

SQP method is adopted to solve this nonlinear optimal problem, and this method is more efficient and accurate than those direct searching or heuristic algorithms which have been approved by [10]. Though the algorithms of SQP method is complicate and not easily understanding, MATLAB has provided professional SQP optimizer. And in this paper, this optimizer is used directly for solving the optimization function.

4. Experiment results and discussion

To verify the integrated method for ultra-precision freeform surface evaluation, a simulated surface was studied which simulated the measured surface. In this case study, the sinusoidal grid surface was selected as the designed surface which can be expressed as the following equation.

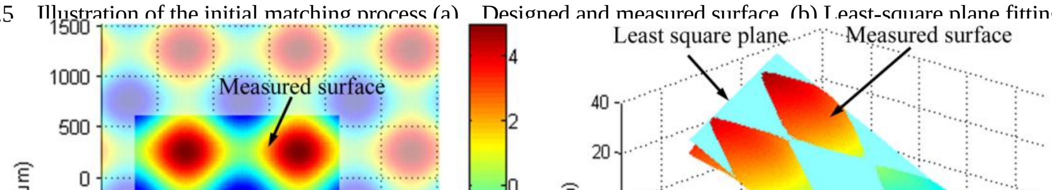
$$z = A_x \sin\left(\frac{2\pi x}{\lambda_x}\right) + A_y \sin\left(\frac{2\pi y}{\lambda_y}\right) - Z_0 \qquad (3)$$

Where, A_x, A_y are the amplitude of the sine functions in the x-direction and y-direction, respectively. λ_x and λ_y are the corresponding spatial wavelengths. Z_0 is a constant offset value. The wave length of the sine functions in x-direction and y-direction are both 1000μm, the amplitude are A_x =2.5μm, A_y=2.5μm, and the constant offset value of Z_0 is 0. Then a portion of the designed surface with the area of −1.2mm≤x≤0.6mm, −1.2mm≤y≤0.6mm was selected to simulate the measured surface as shown by fig.5 (a). In actual situation, measured surface may processes an arbitrary position. So to simulate the measured surface, the selected area of the designed surface was rotated and translated as illustrated by fig.5 (b). The first step is to fit the freeform surfaces using least square plane, so as to build a relatively consistent base plane for both the measured and designed surfaces. While, if we merely select the least square plane as the base plane, there might be exist a great variance between the base plane of designed and measured surfaces. This can be illustrated by fig.3 with the changing of the selected area the least square plane changed. So the selection of the plane paralleling with the least square plane and having a given distance to the specific point like the peak points as the base is more reasonable.

In this case study, the base planes are those paralleling with the least square planes and having the distance to the peak points of 5μm. And the cutting plane is above the base plane with the distance of 1.5μm. Then the intersection lines of both designed surface and measured surface can be obtained as shown by fig.5 (c) and fig.5 (d). Feature points with the maximum curvature are also signified in each figure. To each of the closed intersection curve, there are four feature points. And the distribution pattern of the feature points is of quite similar.

Initial matching of measured and designed surface then can be achieved by the registration of feature points. The designed feature points (DFPs) are in designed frame, and the measured feature points (MFPs) are in measured frame as shown by fig.6 (a). It is quite interesting to note that the registered feature points (RFPs) locate quite close with the designed feature points and a well registration result has been achieved. Fig. 6 (b) is the initial matching result of the simulated and designed surface, from this figure it can be seen that the peaks and valleys are well fitted. By calculating the distance between each point on the measured surface and the designed surface the form error after the initial matching can also be obtained with the maximum value of 0.98μm and the minimum value of −1.42μm, as shown by fig.6 (c). This also indicates a good initial matching result has been achieved. In ideal situation, the final evaluated form error should be zero. While, in actual situation, due to the calculating error of point to surface distance iteration process, the rotation and

Fig.5 Illustration of the initial matching process (a) Designed and measured surface (b) Least-square plane fitting



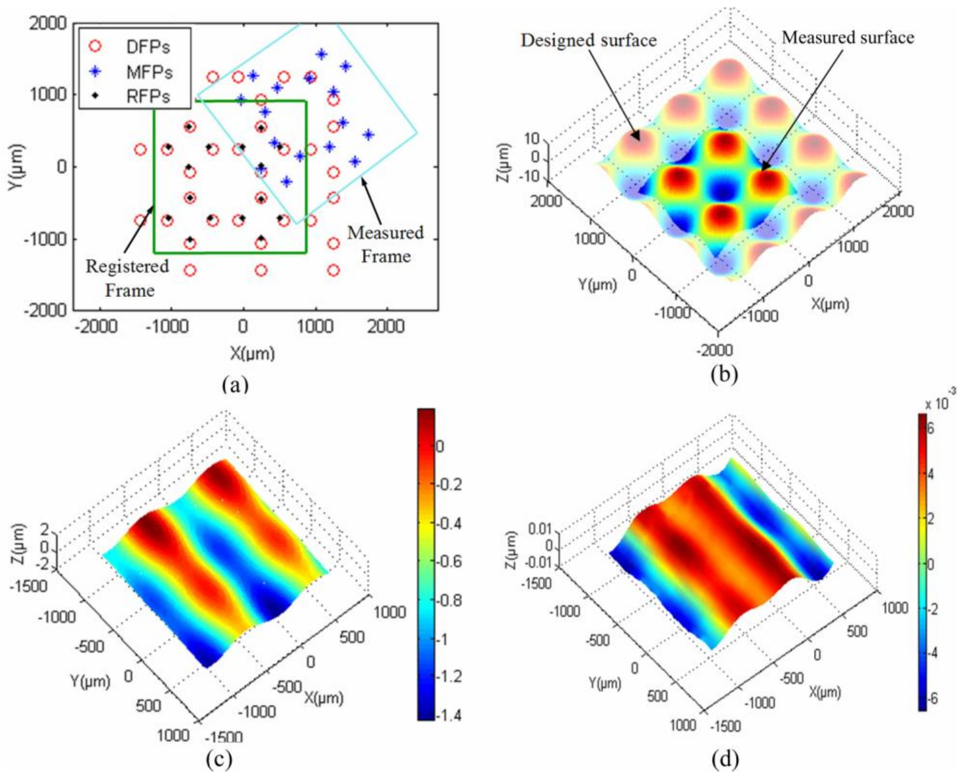


Fig.6 Freeform surface matching and the form errors (a) Point pattern registration, (b) Initial matching result (c) Form error after initial matching, (d) Form error after fine matching

translation of the measured surface, and the optimal algorithms, final matched results as illustrated by Fig.6 (d) could not be zero. The final error value can be considered as the accuracy of this integrated evaluation system. From fig.6 (d), it also can be seen that the maximum error is 6.37nm and the minimum error is −6.40nm. This indicates the proposed freeform evaluation method can reach a precision of nanometer range and capable for evaluation of ultra-precision freeform surfaces.

4. Conclusions

An integrated method for evaluation of ultra-precision freeform surface is presented in this paper which can be generally divided into two steps means the initial matching step and fine matching step. Previous of the two matching steps, preprocessing is needed to separate the roughness surface from the measured surface such that a relative smooth surface can be achieved. The initial matching is a key step for the ultra-precision surface evaluation. From the initial matching results is can be concluded that the proposed dimension-reduced method can be achieved by just registering the very limit of the feature point is of quite efficient and accurate. This method very suited for initial matching of complicated freeform surfaces and can provide a relative good initial searching point for fine matching. The SQP algorithm can adjust the measured surface to well match with the designed surface from micrometer level to the nanometer level. The verification process by use of the simulated surface shows that this integrated method can reach a matching precision of nanometer range and is suited for evaluation of the ultra-precision freeform surface.

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